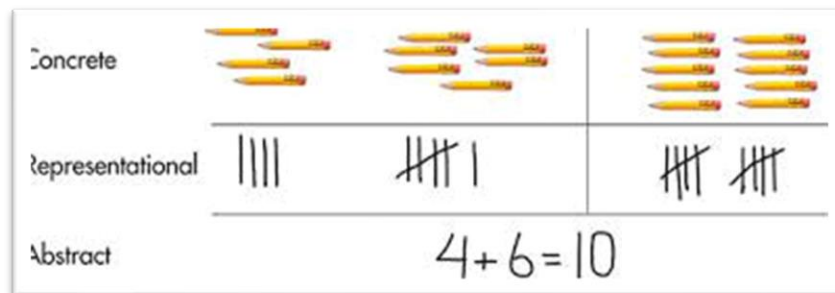


Balanced Math : Conceptual Understanding CRA Continuum

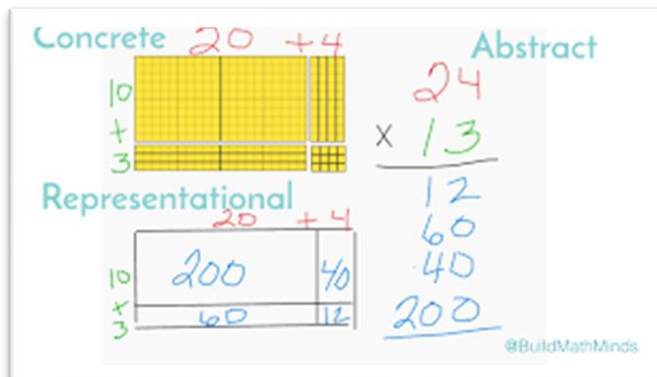
"Balanced Math" means procedural fluency is preceded by conceptual understanding. The two go hand in hand and each supports the other. Procedural fluency (memorizing facts and regurgitating procedures) without conceptual understanding does not require mathematical understanding.

The movement through introduction of concepts in real life (counting, measuring, collecting data, grouping, balancing), followed by representations (pictures, models, drawings, diagrams, graphs) and then finally to abstract mathematical procedures (equations, calculations, generalizations, algorithms) is called the "Concrete-Representational-Abstract Continuum" (or, Concrete-Pictorial-Symbolic, as in our curriculum).



<https://iris.peabody.vanderbilt.edu/module/math/cresource/q2/p05/>

<https://www.therecoveringtraditionalist.com/concrete-representational-abstract-approach/>

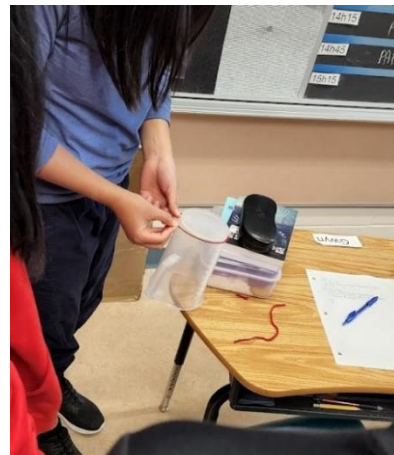


Concrete step of CRA

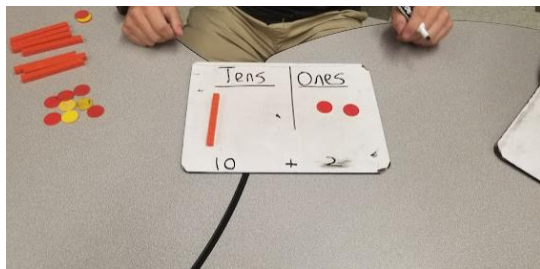
Concrete is the "**doing**" stage. During this stage, students use concrete **objects** to model problems. Unlike traditional maths teaching methods where teachers demonstrate how to solve a problem, the CPA approach brings concepts to life by allowing children to experience and handle physical (concrete) objects. With the CPA framework, every abstract

concept is first introduced using physical, interactive concrete materials.

Grade 7 students at St. Michael's School develop a concept of Pi



Balanced Math : Pictorial/Representational CRA Continuum



SSST at St. Mary's helps students understand place value using concrete materials

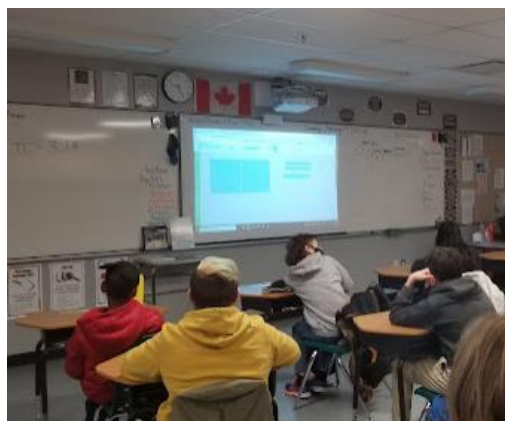


Grade five students at St. Paul's exploring division, factors, and prime numbers with concrete materials

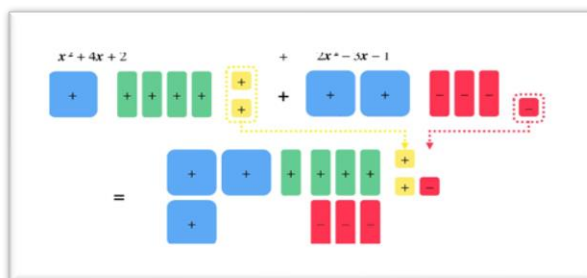
Pictorial (representational) step of CRA

Pictorial or Representational is the "seeing" stage. Here, **visual** representations of concrete objects are used to model problems. This stage encourages students to make a mental connection between the physical object they just handled and the **pictures**, **diagrams** or **models** that represent the objects from the problem.

Building or drawing a model makes it easier for children to grasp difficult abstract concepts (for example, fractions). Simply put, it helps students visualise abstract problems and make them more accessible.

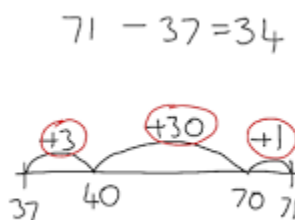


Grade 7 students at St. Michael's view a picture of base ten blocks representing decimal numbers. This is a *model* of a decimal number.



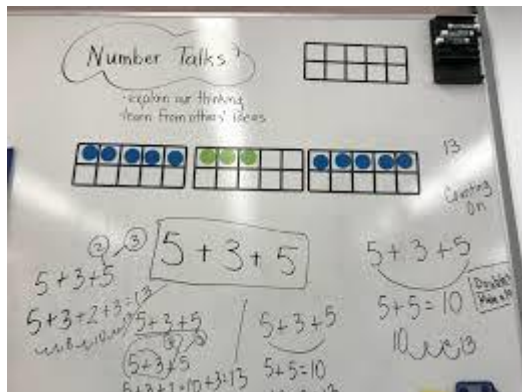
<https://www.slideserve.com/saad/algebra-tiles-can-be-used-to-model-polynomials>

A model or representation of polynomial addition.



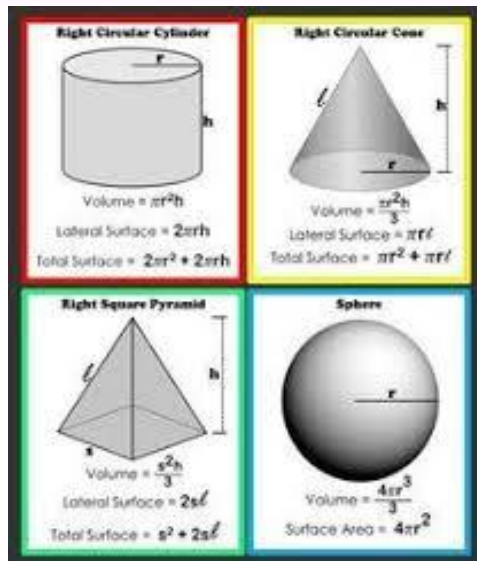
An open number line used to represent thinking around subtraction.

CRA Continuum



Representations recorded during a number talk

<https://shelleygrayteaching.com/concrete-representational-abstract-model/>



https://s-media-cache-ak0.pinimg.com/236x/67/4c/49/674c497130441912fbd24bef26474906.jpg

These volume and surface area diagrams are 2 dimensional *representations* of 3 dimensional figures. The formulas beneath would be the abstract step of the continuum

Abstract step of CPA

Abstract is the “symbolic” stage, where students use abstract **symbols and algorithms** to model problems. Students will not progress to this stage until they have demonstrated that they have a solid understanding of the concrete and pictorial stages of the problem. The abstract stage involves the teacher introducing abstract concepts (for example, mathematical symbols). Children are introduced to the concept at a symbolic level, using only numbers, notation, and mathematical symbols (for example, $+$, $-$, $\times$, $/$) to indicate addition, multiplication or division, using algorithms or procedures to derive a solution.

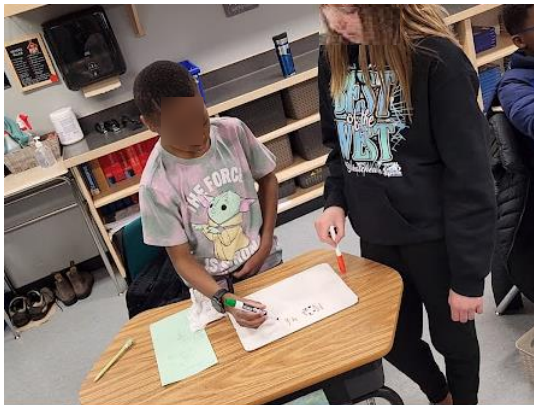


Grade 7 students at St. Alphonsus' School play Fraction Formula, which connects the concrete fractions to symbolic (abstract) fraction.

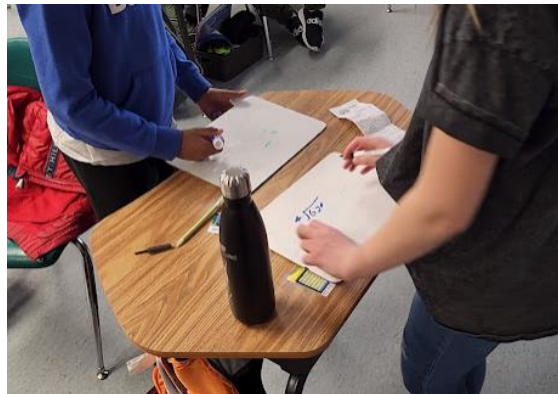


Students at St. Michael's School connect concrete numbers on a rekenrek to abstract computations and representations

Balanced Math : Procedural Fluency/automaticity CRA Continuum



Students at St. Michael's School practice abstract algorithms—adding fractions



Students at St. Michael's School practice abstract algorithms—long division

Procedural Fluency or automaticity comes from meaningful practice. This includes not only independent practice, but mathematical dialog, collaboration, peer teaching, self-assessing, descriptive non-graded feedback, number talks and numeracy routines, use of math programs such as Mathletics and IXL, repetition, and conferring with the teacher. Procedural fluency is supported by taking care to make connections in math—both between the concrete and pictorial models and the algorithms, but also between concepts in math (Ex: fractions, decimals, percents, ratio, and division) and between math and real-world contexts.

Higher grades involve more abstract thinking and computation:

Students still benefit from seeing models (representations) or concrete examples, so they understand what is happening in the procedure and what we are actually finding:

Solving Quadratic Equations using Completing the Square

Get all the x terms on the left and c numbers on the right

Divide by coefficient of x^2 to get $a=1$
Complete the square. To keep equation balanced, add $(b/2)^2$ to both sides

Write as a binomial squared.

$$\left(x + \frac{b}{2}\right)^2$$

Take square root of both sides and solve for x

$$2x^2 - 12x + 9 = 5$$

$$2x^2 - 12x = -4$$

$$x^2 - 6x + 9 = 27 + 9$$

$$(x-3)^2 = 36$$

$$x-3 = \pm 6$$

$$x = 3 \pm 6$$

$$x = 9, -3$$

<https://byjus.com/maths/completing-the-square/>

