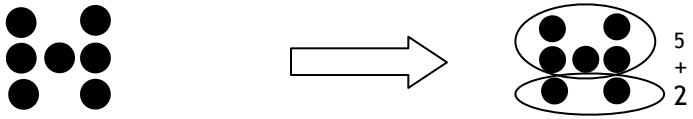
Blackline Master #1 - Dot Pattern Postcards

You can print these on cardstock and then cut into the four cards per page. However, the format was designed to fit on the PostCard template paper you can buy at office supply stores (I used Avery® template 8387TM). The pages come perforated so that once you print all you have to do is tear them into the four cards.

There are many uses for dot pattern cards beyond the activities listed in the book (see ideas below), but the main goal when using dot patterns is to encourage children to determine how many they see *without counting one-by-one*. These dot patterns are arranged on purpose to help children see *groups of dots* instead of individual dots. As children progress with their mathematical development, it is essential that they be able to see groups that come together to make a whole AND to see that a whole can be broken into groups. For many children the only picture they have of seven is the numeral. That "picture" makes it almost impossible for them to see how that can be broken into a 5 and a 2, or a 6 and a 1. However if a child sees this for seven, they can actually see a 5 and a 2:

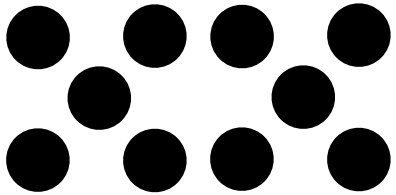


Using that same pattern, some children may see the 5 and 2 differently:

And even cooler is that they do not have to see it as only 5 and 2. Some may see 6 and 1 or 4 and 3 (Can you see them?). The point is to give them a visual representation of the abstract symbols we ask them to compute with. Many children are still grappling with making sense of single digits; that 7 can be 7+0, 6+1, 5+2, 3+4, and so on. However at the same time they are dealing with this understanding we are asking them to think about combining those same numbers but with place value: 1 and 6 now become 16. When we give them a visual picture of 1 + 6 they can see for themselves that it makes 7 and not just because we put a '+' sign in between them.

Building these spatial relationships also builds the idea of subitizing. Subitizing is being able to instantly recognize how many are in a set (Clements, 1999). For most people we can only subitize small sets (less than 5) unless they are in some kind of familiar pattern. For larger sets of numbers we tend to decompose the set into smaller sets that we do instantly recognize. Look at the dots below and determine how many total dots there are:

Did you do it without having to count every single dot to make sure of your total? Now try this one:



With this pattern you can instantly recognize the familiar pattern of five and there are two of them, so there has to be ten.

Possible activities:

Make the Pattern: provide each student with about ten counters and a piece of construction paper as a mat. Hold up a dot pattern for about 3 seconds and have the children make the pattern they saw using the counters on the mat. *"How many dots did you see? How did you see them?"* Spend time discussing the configuration of the pattern and how children saw how many dots there were.

Overhead Pattern: print the templates on overhead transparency flash one pattern for about 3 seconds. Have a blank transparency over the dot pattern/ten frame/MathRack and ask students to come up and show how they figured out the number shown. Encourage them to circle the groupings they saw.
Which of these is not like the other? show three cards, all but one of which have the same number just in a different way. Children have to find the one card that does not belong.

Flashcards: hold up a card for 1-3 seconds. *"How many were shown on that card? How did you see it?"* Include lots of easy patterns to build up their confidence. This can also be done with partners as a workstation activity.

Fast Flash: have two students stand on opposite sides of an overhead projector. One student holds a pack of cards printed on transparency paper. The student places one of the cards on the projector and the other student takes it off as fast as she or he can. Then the members of the class, along with the teacher, race to announce the number that was shown.

Can you find it? Grab two cards of each number 1-10 (since there is only one card for the 1 you need to print an extra of that card so that you have a two 1 cards, or do an additional two cards of another number). Place the cards face down randomly in a 4x5 array. For each child's turn they flip over two cards. If the cards show the same amount they get to keep those two cards. If not, they flip the cards back over and the next child gets a turn.

Go Fish: Print off at least two sets of the cards and play using the normal rules of Go Fish.

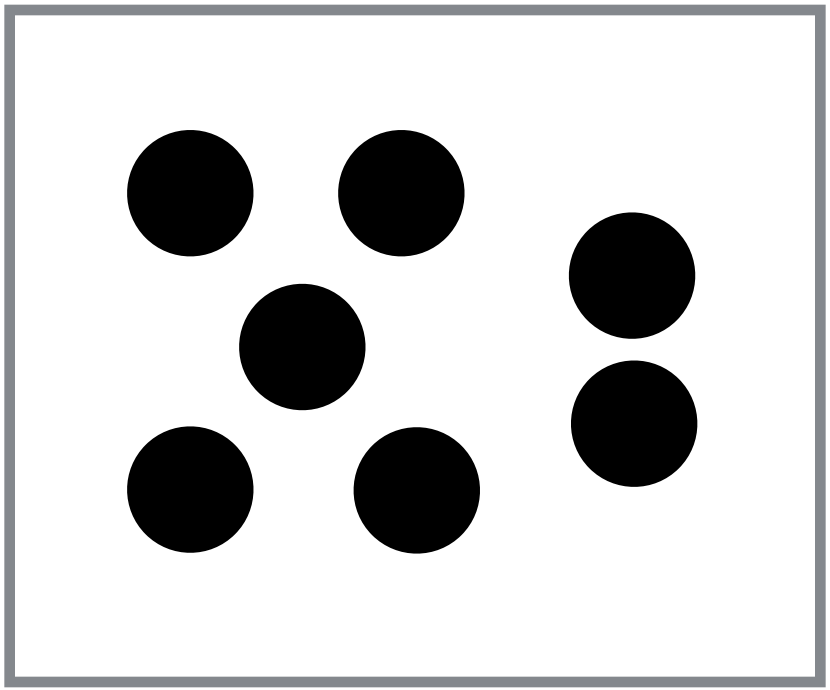
Which is More? Flip over two cards and then ask, "Which card has more?" Once children get better at subitizing you can flash two cards for a few seconds and then hide them before you ask "Which card had more?"

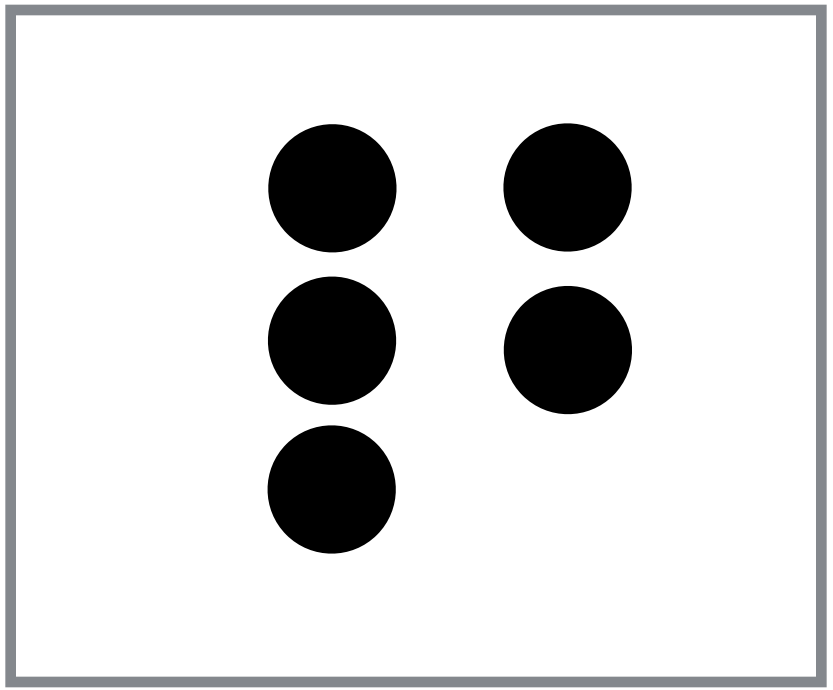
Which is Less? Flip over two cards and then ask, "Which card has less/fewer?" Once children get better at subitizing you can flash two cards for a few seconds and then hide them before you ask "Which card had less/fewer?"

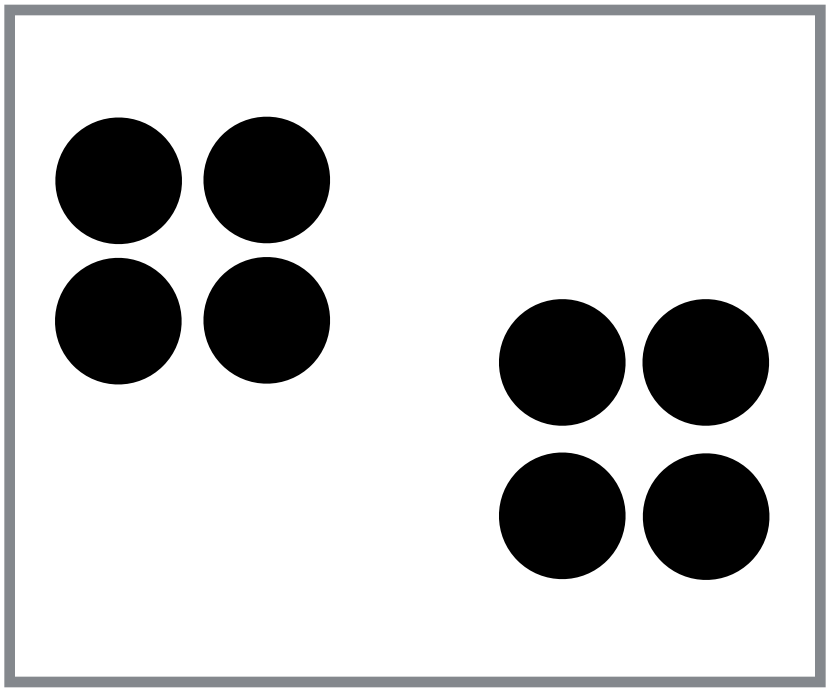
Make it More/Less: Like the "make the pattern" activity, but instead of making the same amount that you flashed tell them to "Make it More than the amount on my card" or "Make it Less than the amount on my card." Or even have them do a set that is More, a set that is Less, and another set that is the Same all at the same time.

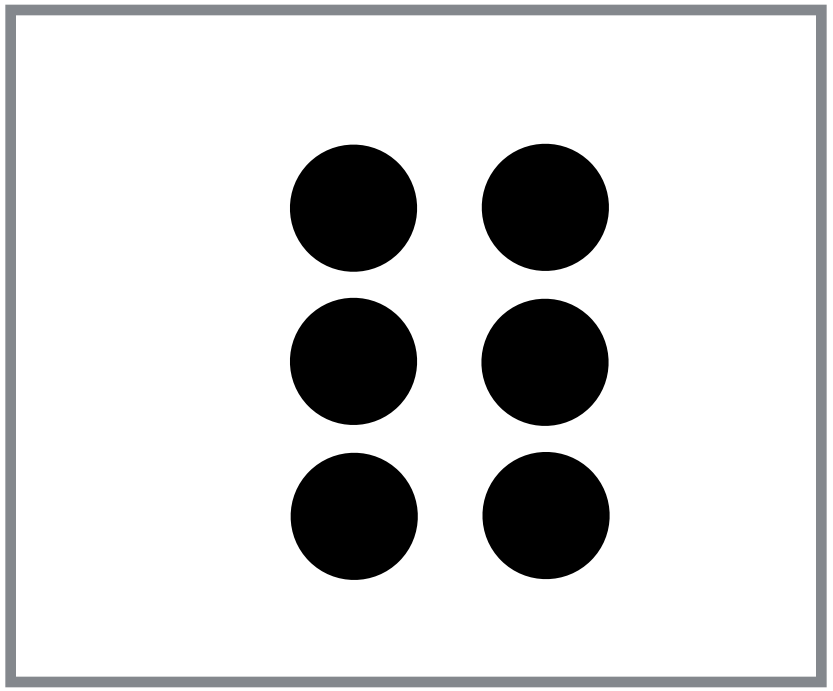
Combat: Two children take the entire deck of subitizing cards and deal them out face down between the two of them, so that each child has an equal amount in a pile. At the same time, each child flips over their top card. The child with the larger amount on their card gets to take both cards. If they flip over cards with the same amount, they each flip over another card to see who has the larger amount and then that player would take all the cards flipped over. Play continues until one player gets all the cards.

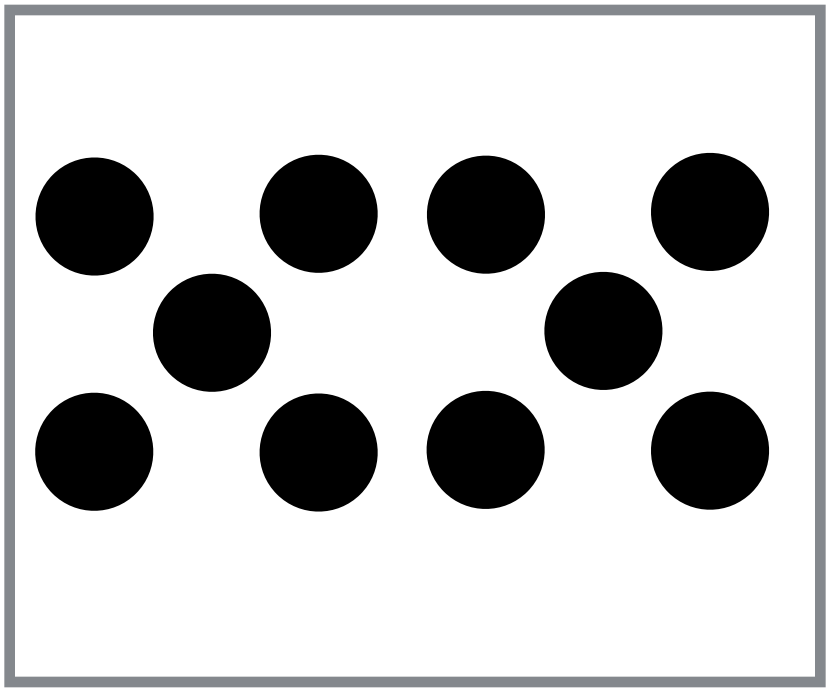
Combat Addition: Played like Combat (above) but instead of flipping over only one card, each player flips over two cards and then adds the amount on the two cards. The child with the larger total on their cards gets to take all four cards.

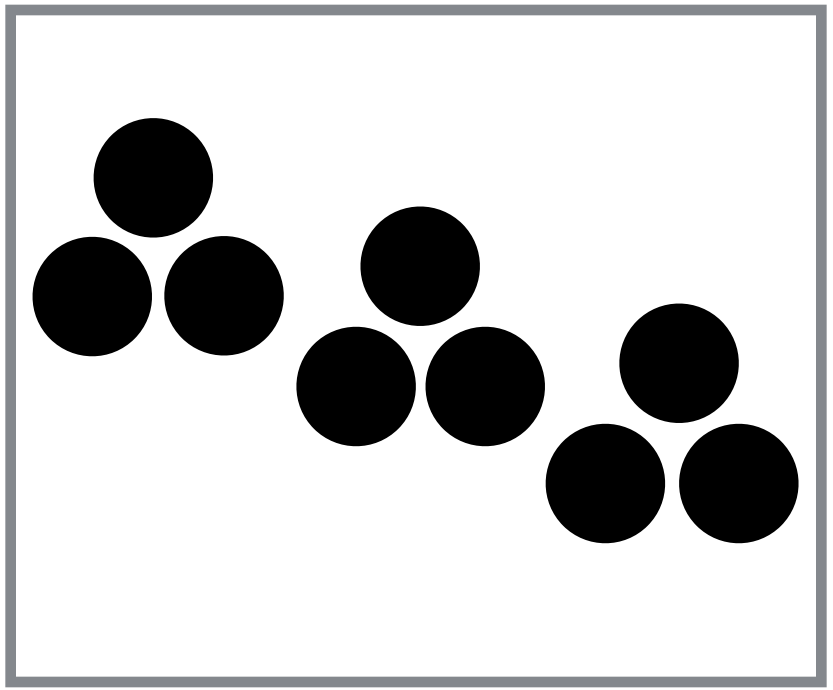


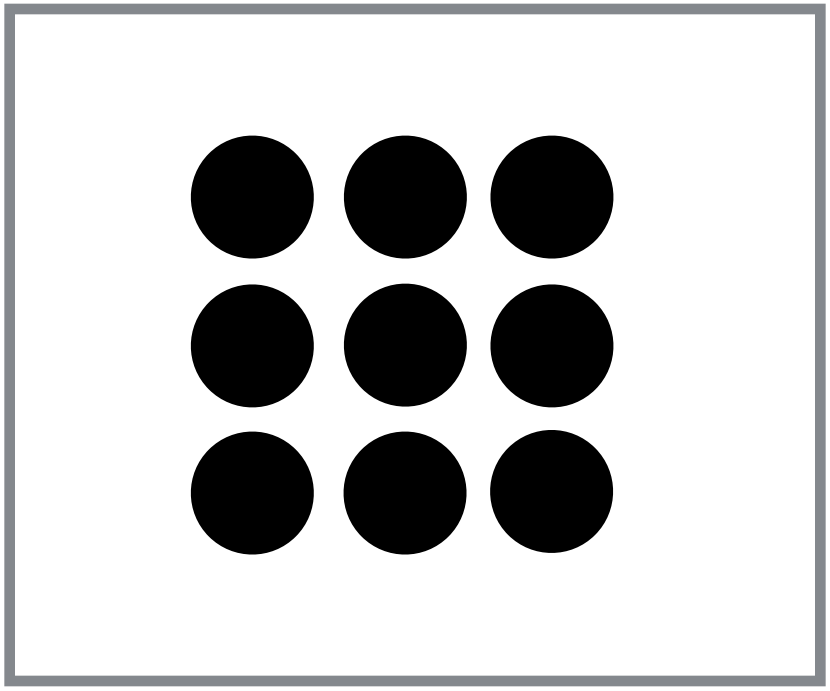


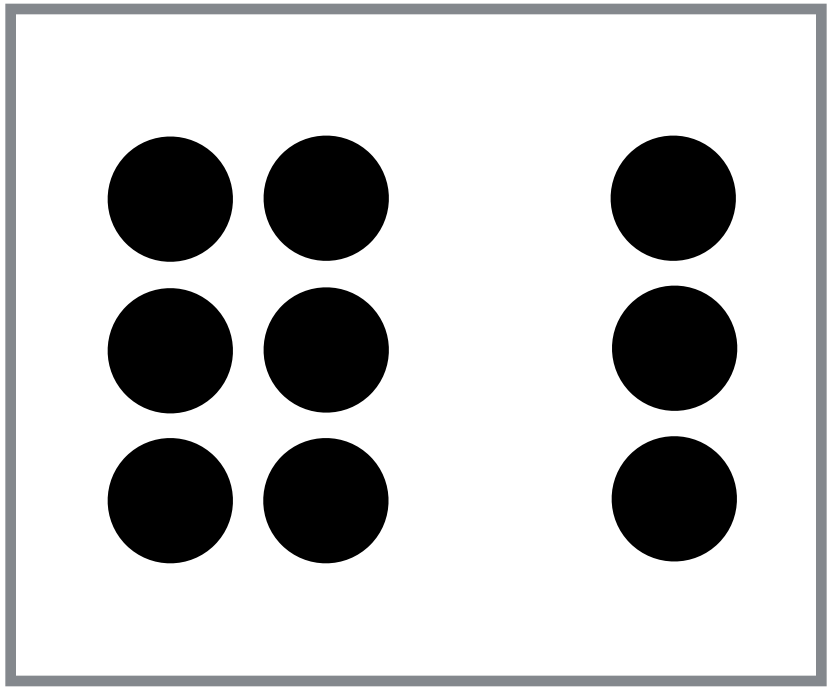


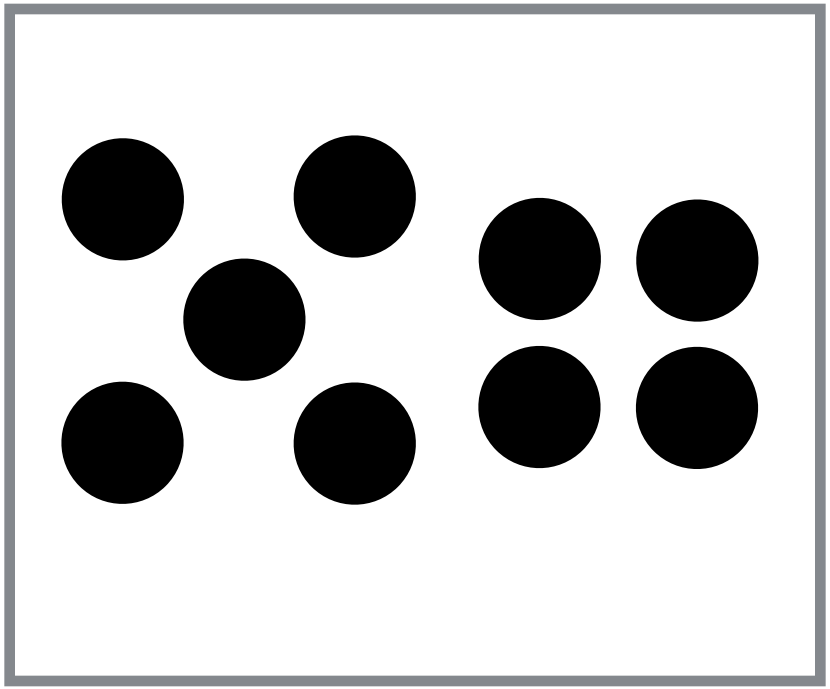


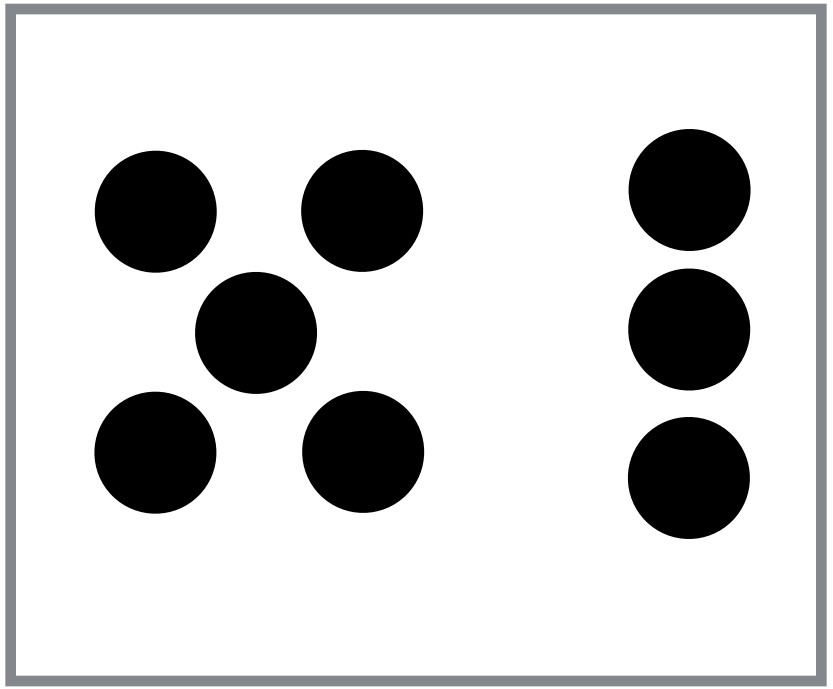


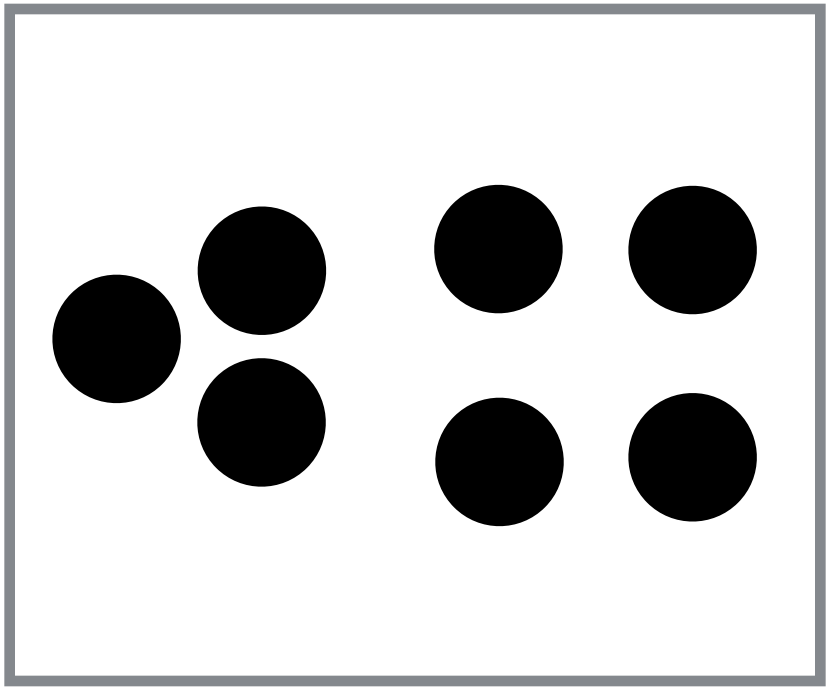


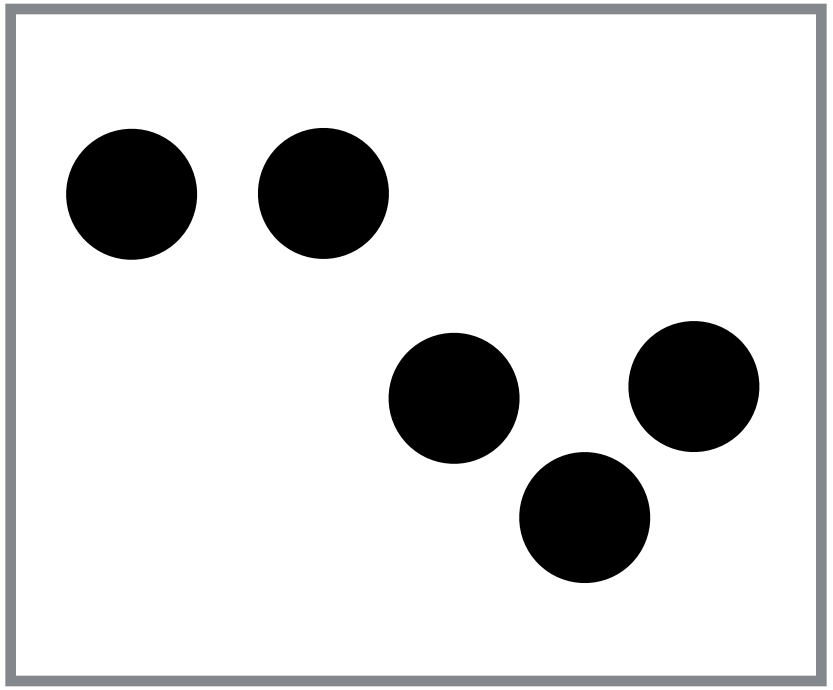


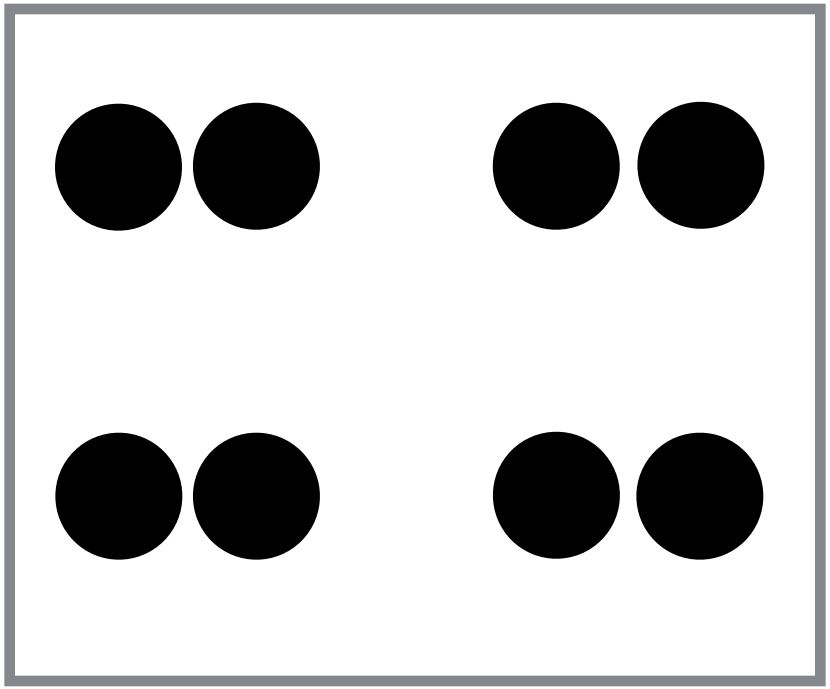


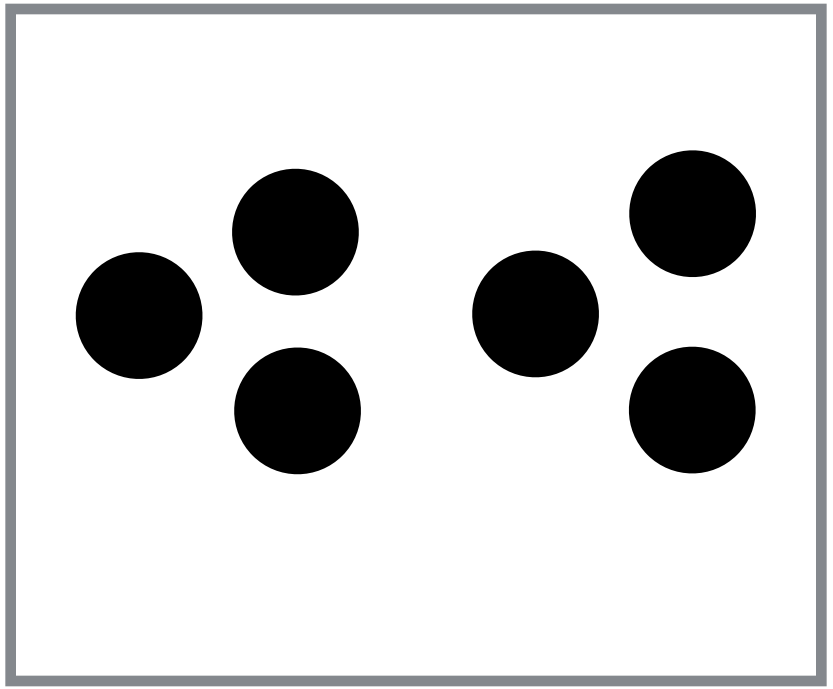


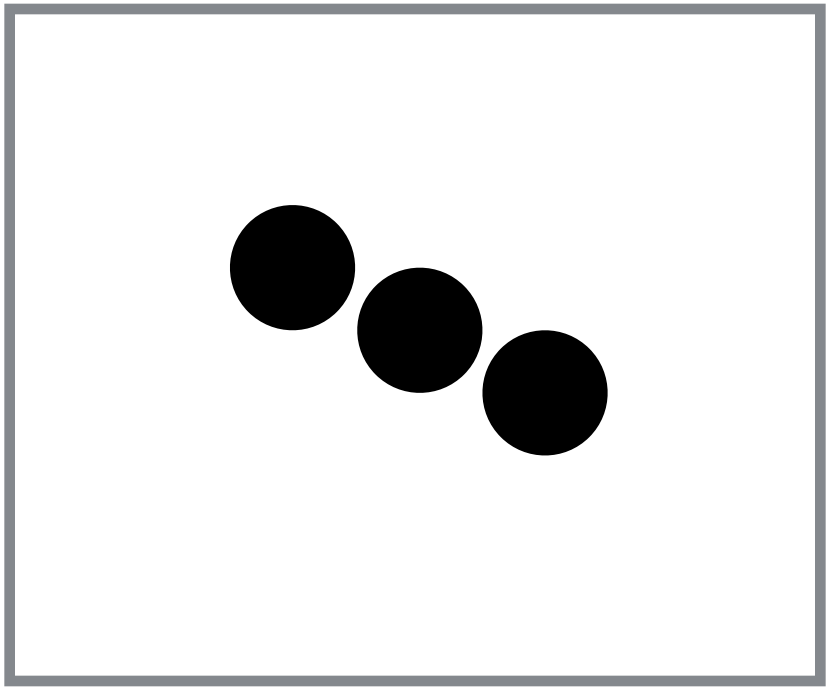


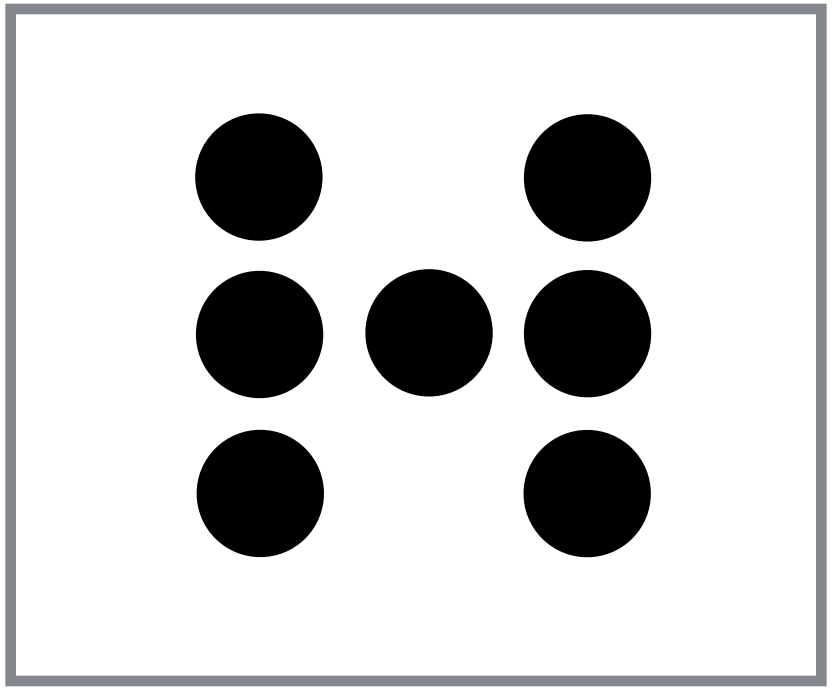


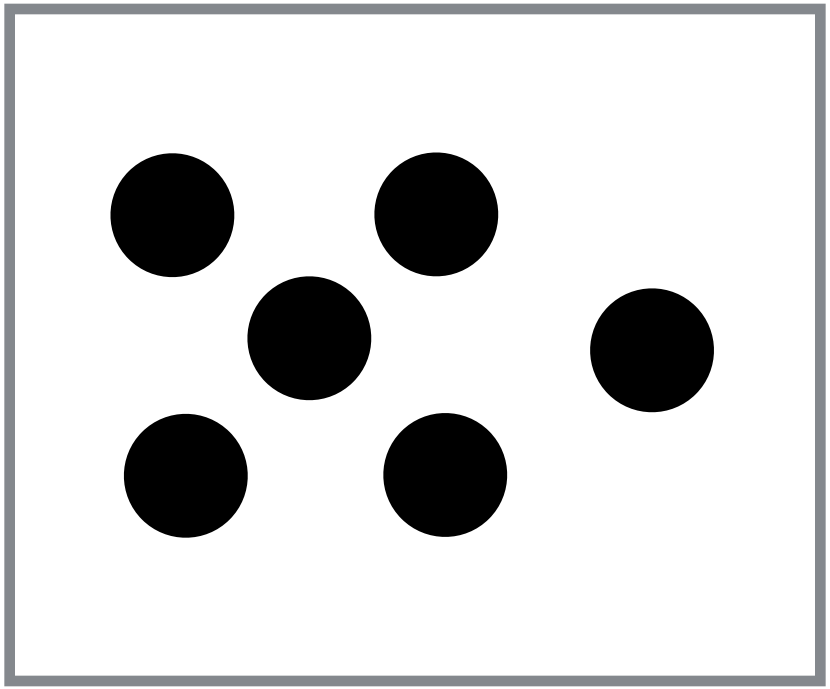


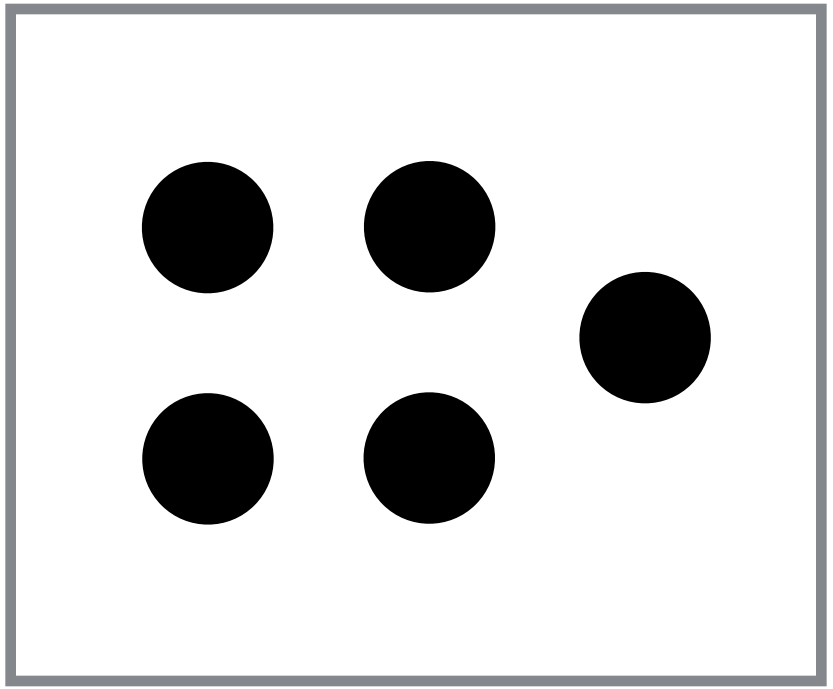


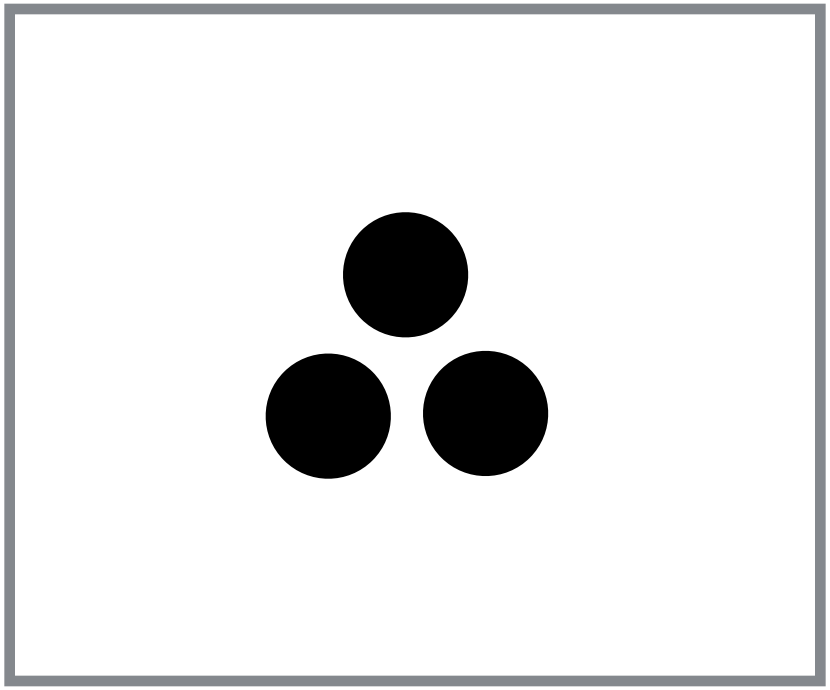


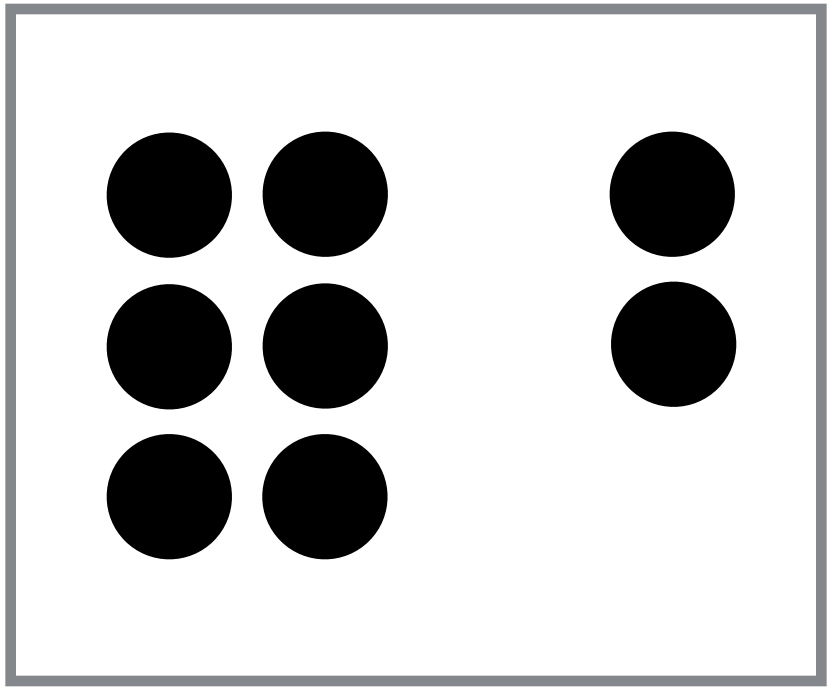


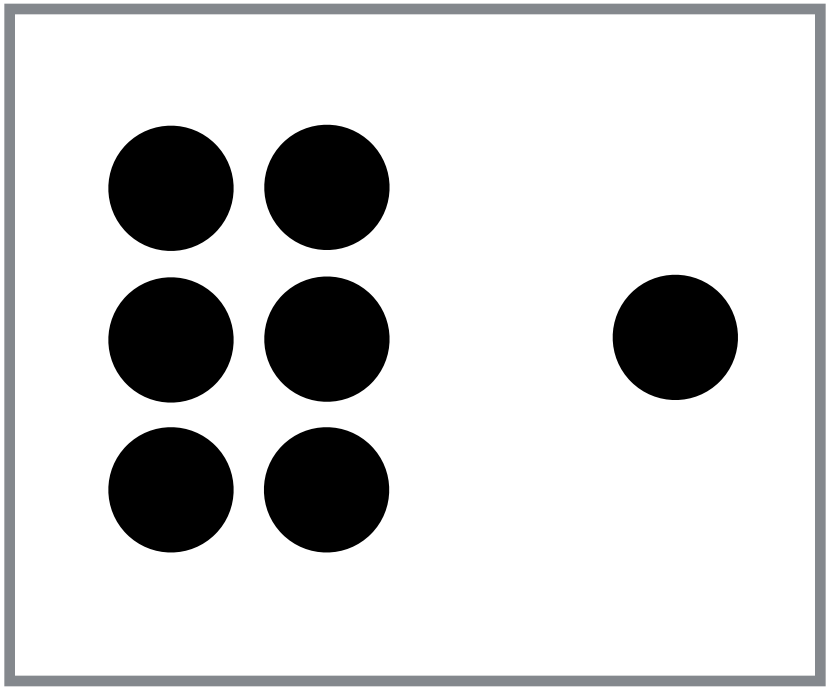


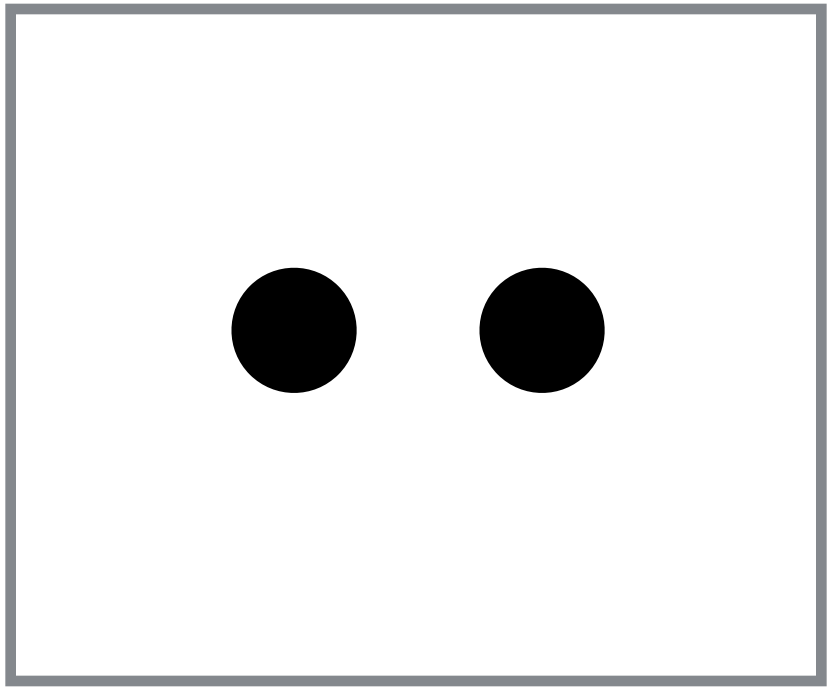


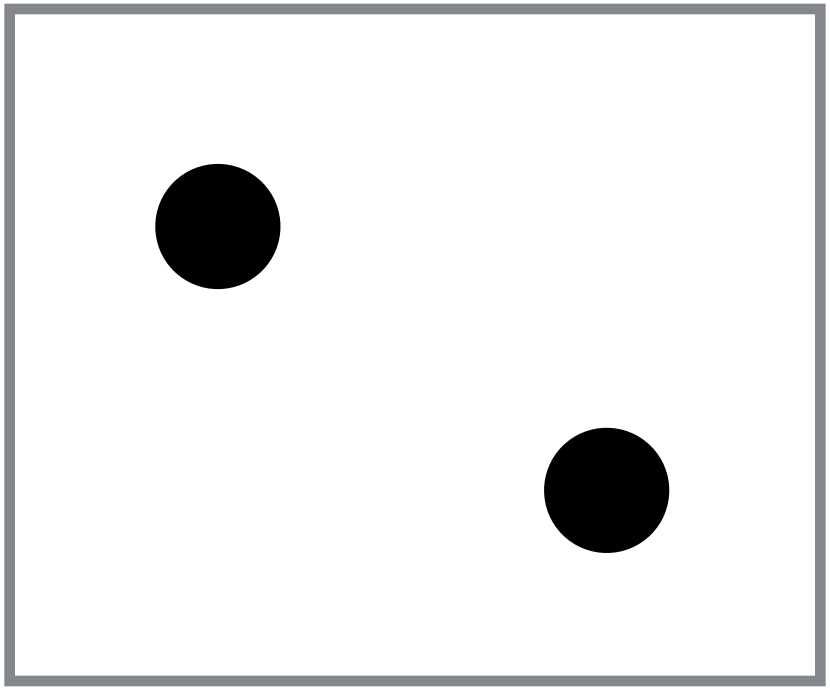


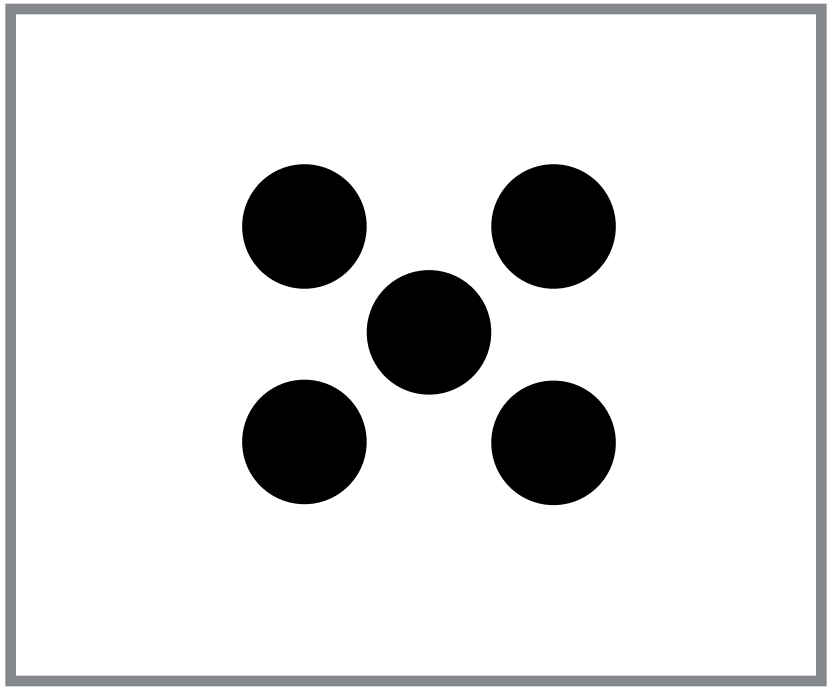


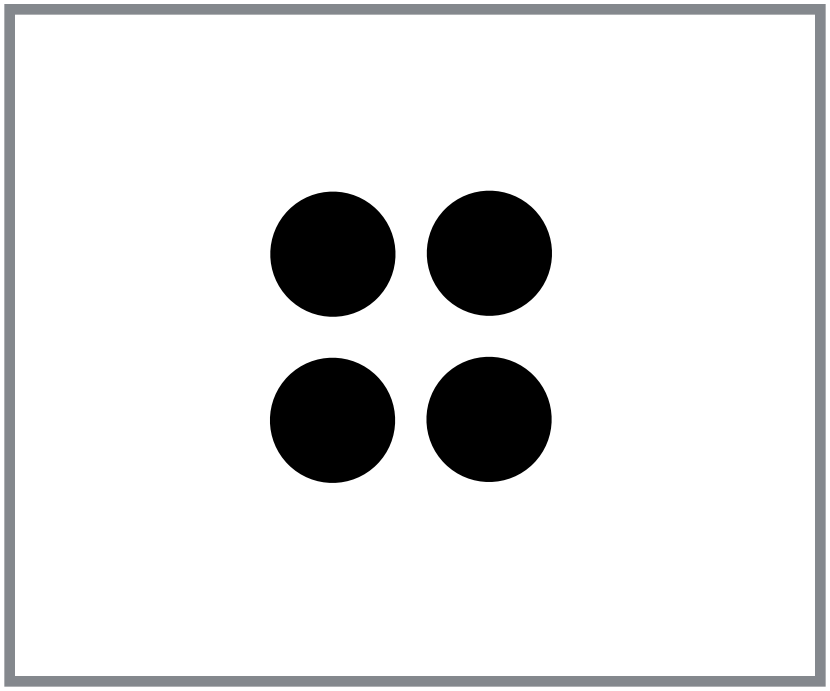


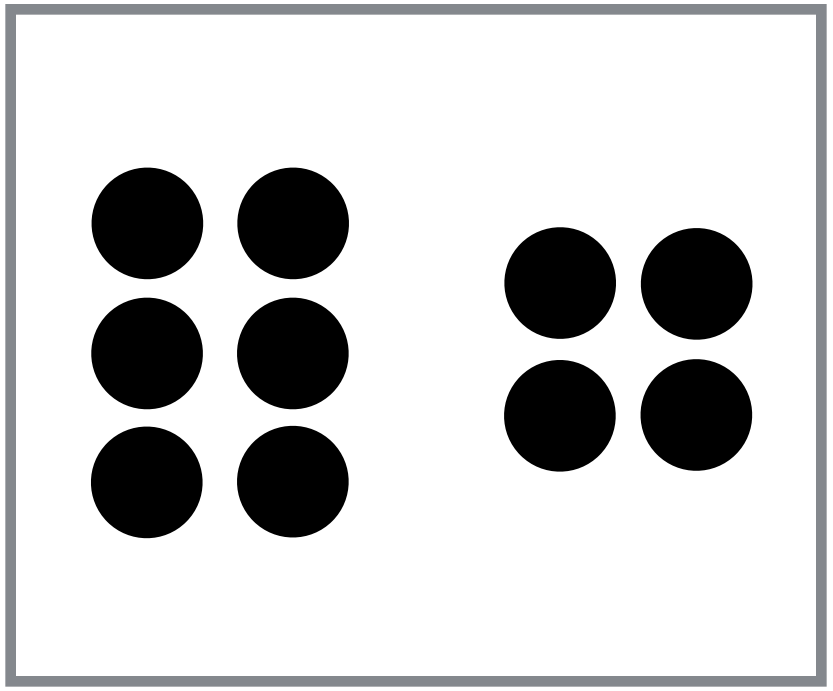


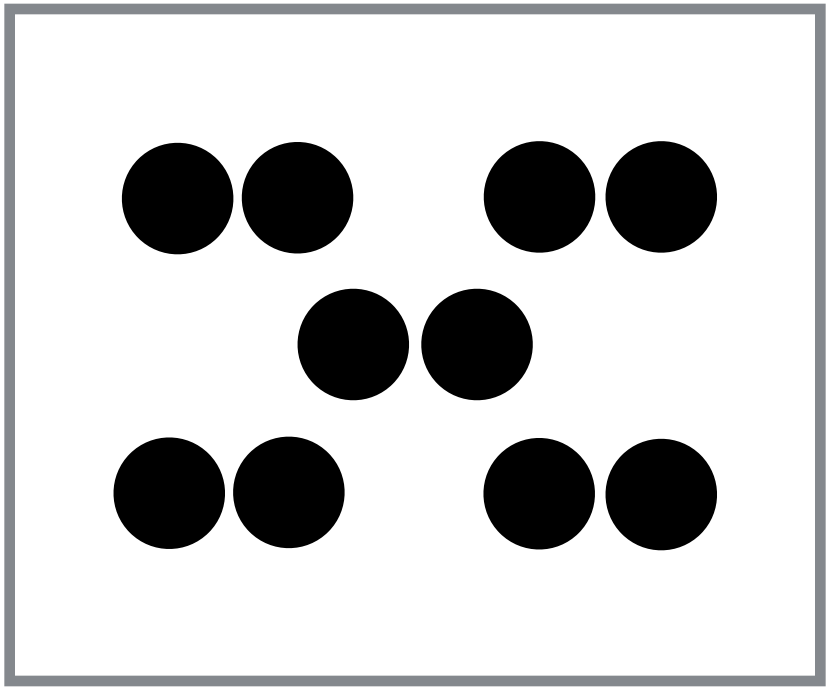


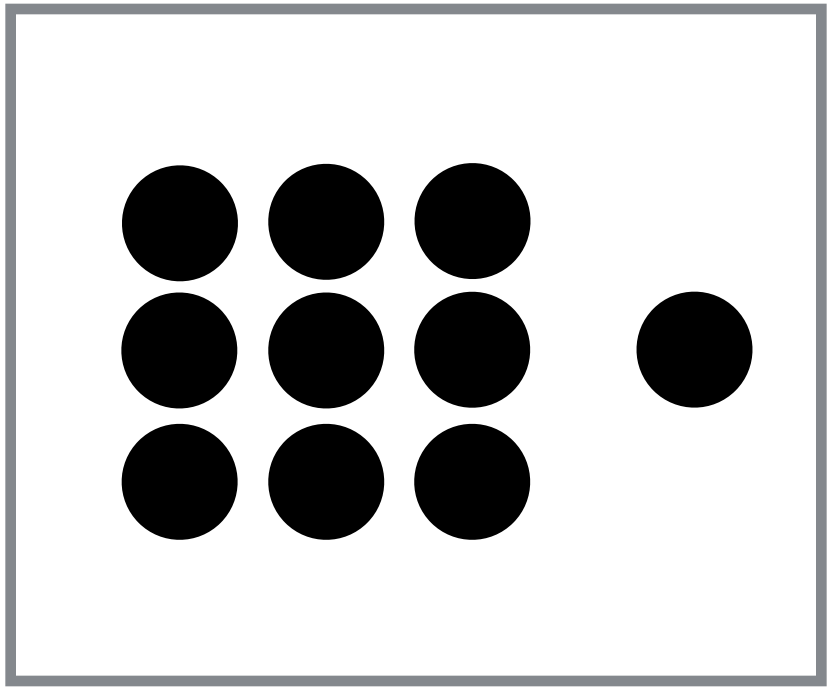


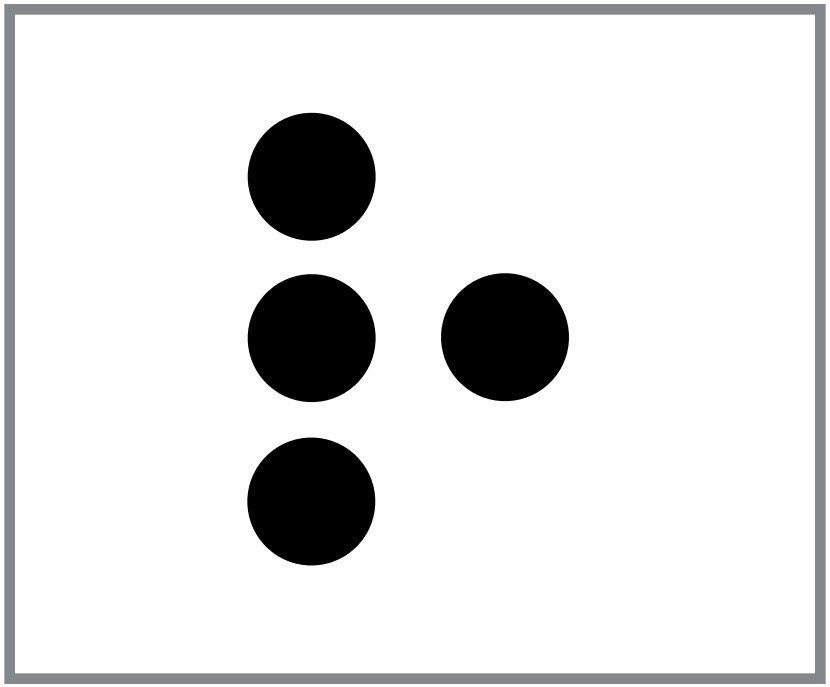


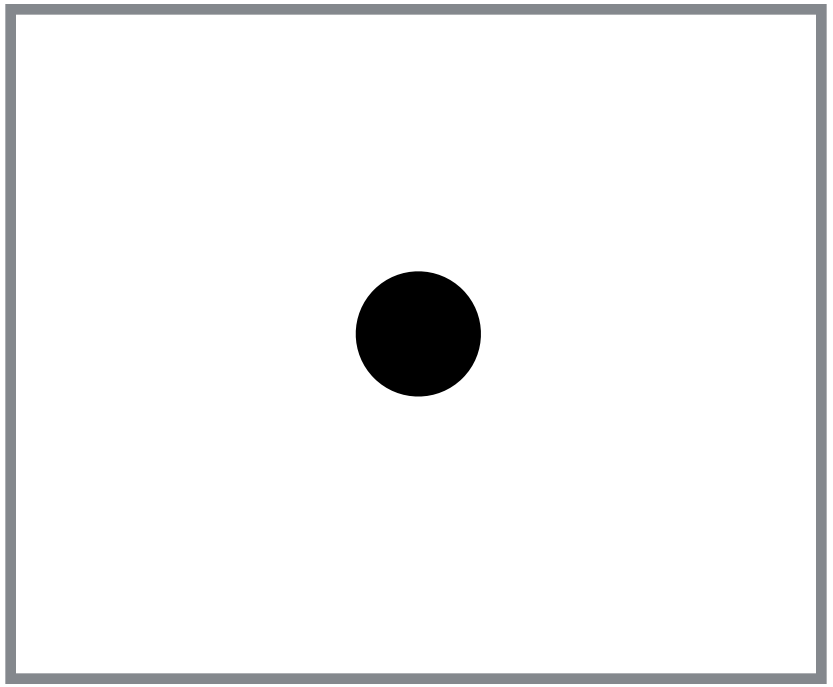


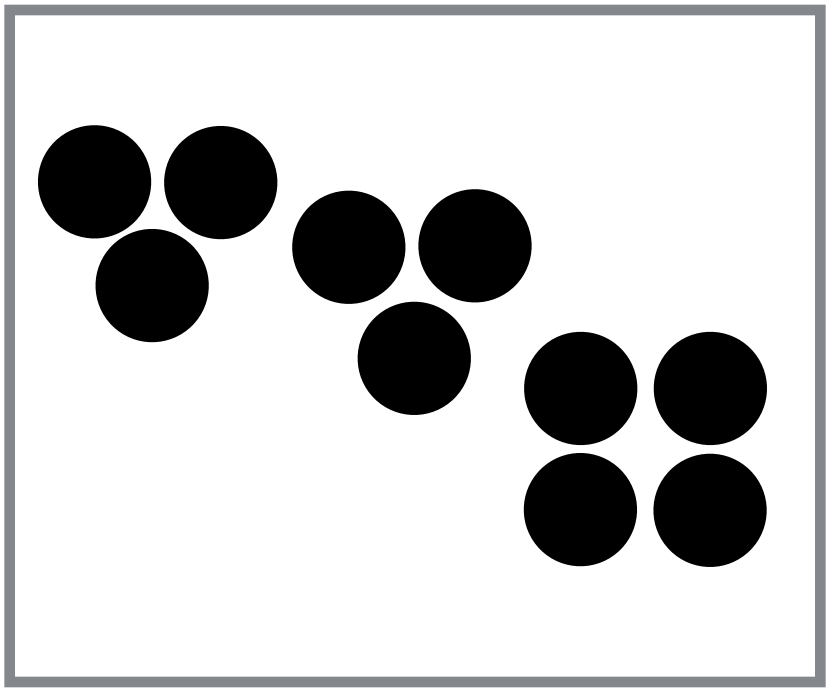


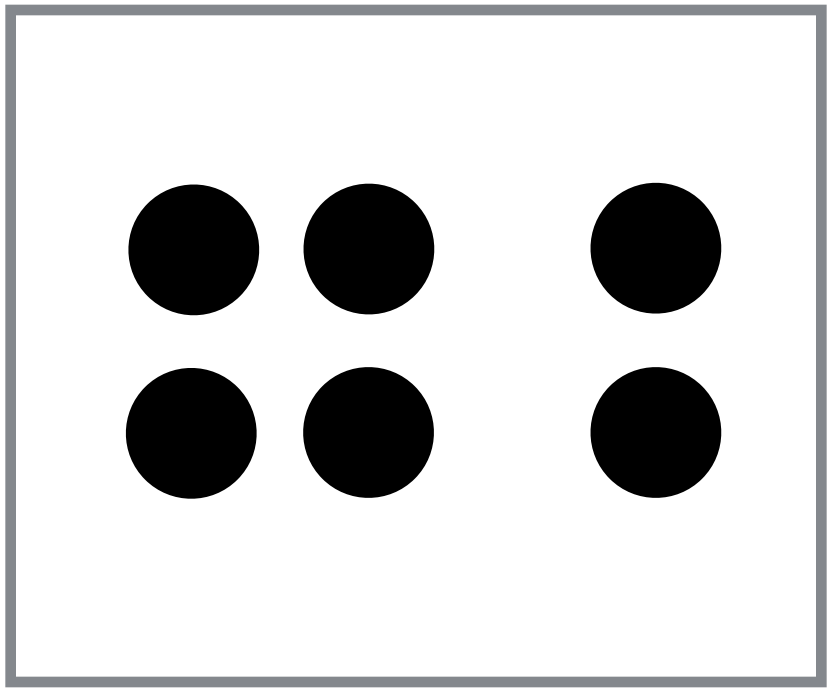






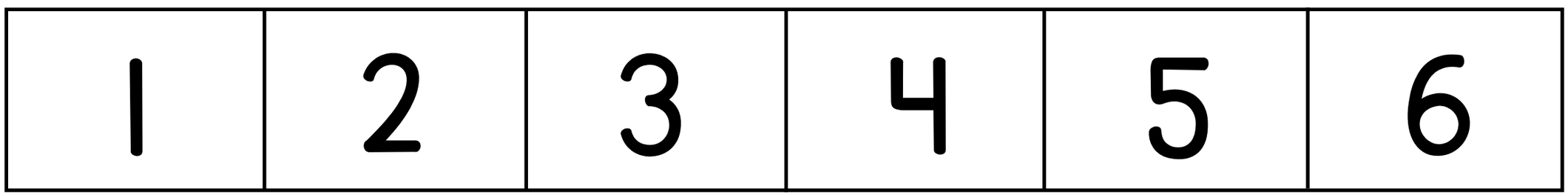


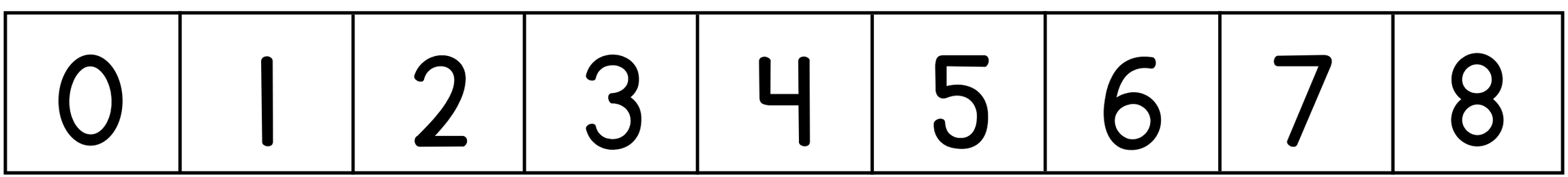




Blackline Master #2 - Cover It Strips

These strips are designed to be printed on 11x17 paper, but they can be printed on 8.5x11, the strips will just be smaller.





Blackline Master #3 - Savvy Subitizing Cards

You can print these on cardstock and then cut the cards out. **You can also purchase pre-made decks of Savvy Subitizing cards at** www.buildmathminds.com.

The educators at Mathematically Minded, LLC, believe that in order to build a child's mathematical mind, connections must be built that help show children that mathematics is logical and not magical. **Building a child's number sense helps them see the logic in numbers.** We encourage you to use these cards in ways that build children's sense of numbers in four areas (Van de Walle, 2013):

1) Spatial relationships: recognizing how many without counting by seeing a visual pattern.

2) One and two more, one and two less: this is not the ability to count on two or count back two, but instead knowing which numbers are one and two less or more than any given number.

3) Benchmarks of 5 and 10: ten plays such an important role in our number system (and two fives make a 10), students must know how numbers relate to 5 and 10.

4) Part-Part-Whole: seeing a number as being made up of two or more parts.

These Savvy Subitizing cards were designed to play a card game I call Savvy Subitizing (modeled after the game Ratuki®). **Printing this whole document actually gives you two decks of cards.** Visit this website for full directions on the games: http://www.therecoveringtraditionalist.com/savvy-subitizing-activity/

Here are brief descriptions of some other activities you can do using these cards:

Fast Flash
Use the cards like flash cards by flashing a card for a few seconds and then put the card down so the kids cannot see it anymore. At first you may need to leave the card visible for longer to allow children to count. But remember, the goal is to get children to tell how many without counting (i.e. Subitize). So start doing it faster and encouraging them to visualize what was flashed after you hide the card.

Can you find it?
Grab two cards for each number 1-10. Place the cards face down randomly in a 4x5 array. For each child's turn they flip over two cards. If the cards show the same amount they get to keep those two cards. If not, they flip the cards back over and the next child gets a turn.

Which is More?
Flip over two cards and then ask, "Which card has more?" Once children get better at subitizing you can flash two cards for a few seconds and then hide them before you ask "Which card had more?"

Which is Less?
Flip over two cards and then ask, "Which card has less/fewer?" Once children get better at subitizing you can flash two cards for a few seconds and then hide them before you ask "Which card had less/fewer?"

Which of these is not like the other?
Lay out three cards (two of which have the same amount) and ask the children to find the card that does not have the same amount as the other two.

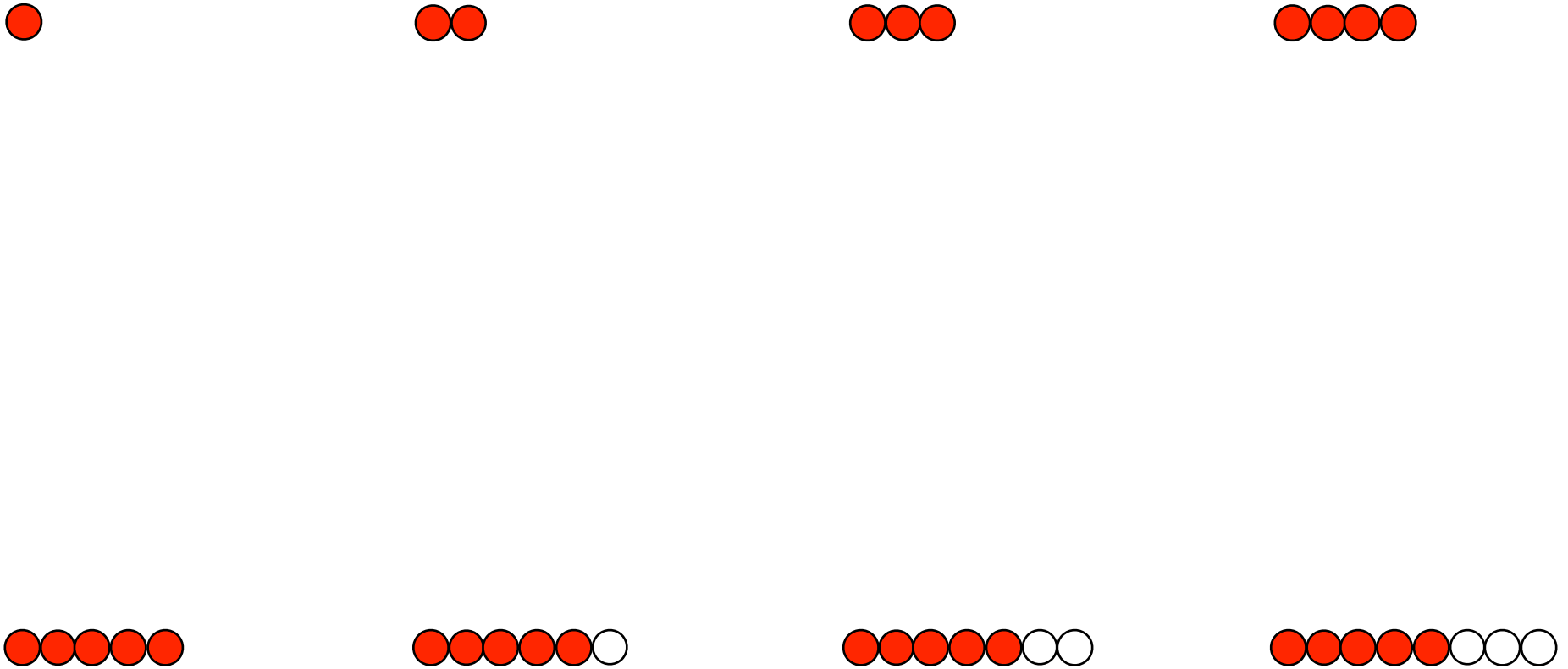
Make it
Flash a card and then have the children recreate the amount.

Make it More/Less
Like the "make it" activity, but instead of making the same amount that you flashed tell them to "Make it More than the amount on my card" or "Make it Less than the amount on my card."

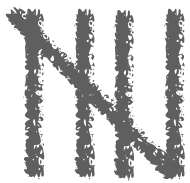
Combat
Two children take the entire deck of Savvy Subitizing cards and deal them out face down between the two of them, so that each child has an equal amount in a pile. At the same time, each child flips over their top card. The child with the larger amount on their card gets to take both cards. If they flip over cards with the same amount, they each flip over another card to see who has the larger amount and then that player would take all the cards flipped over. Play continues until one player gets all the cards.

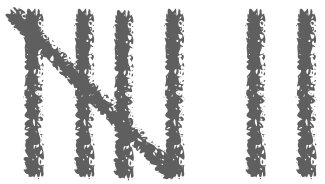
Combat Addition
Played like Combat (above) but instead of flipping over only one card, each player flips over two cards and then adds the amount on the two cards. The child with the larger total on their cards gets to take all four cards. If they flip over cards that total the same amount, they each flip over another two cards to see who has the larger total and then that player would take all the cards flipped over. Play continues until one player gets all the cards.

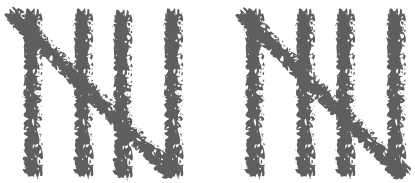
These are just a few activities you can play using these cards. I would love to get feedback on how you use the cards. Either email me at mathematicallyminded@yahoo.com or leave a comment on my blog post about them: http://www.therecoveringtraditionalist.com/savvy-subitizing-activity/





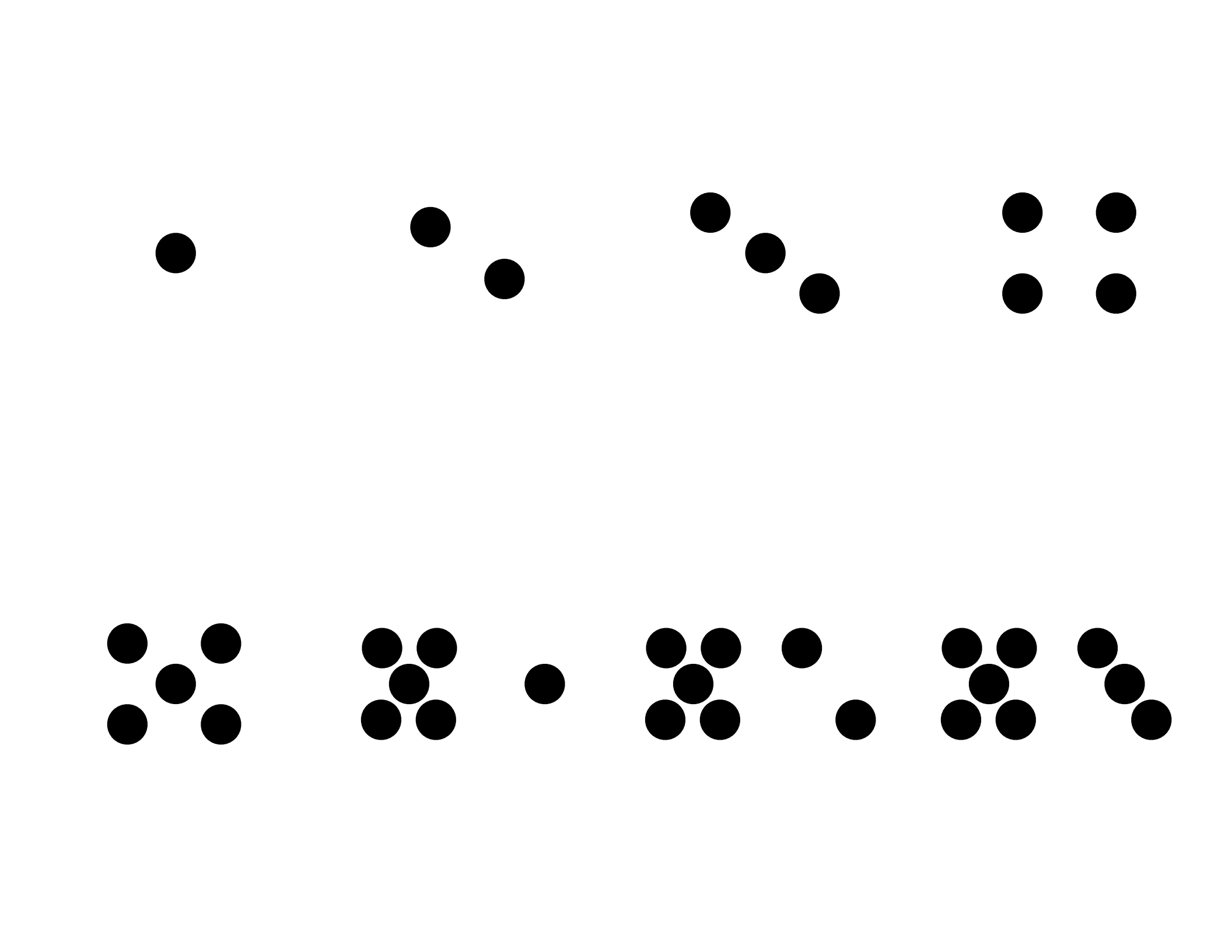



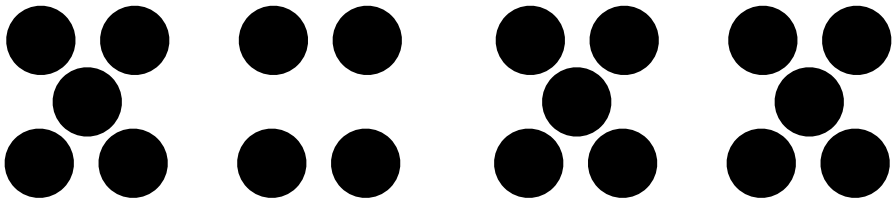


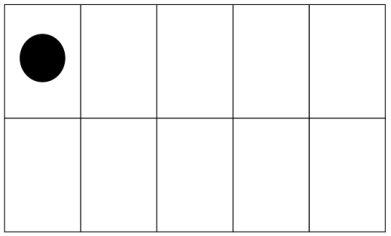


1
2

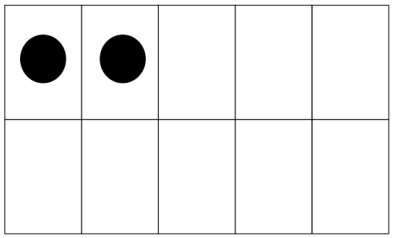
3 4 5 6

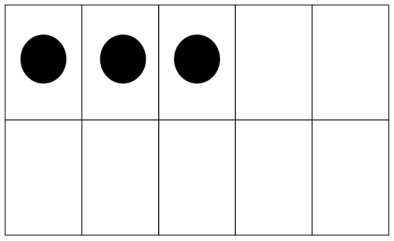
7 8 9 10

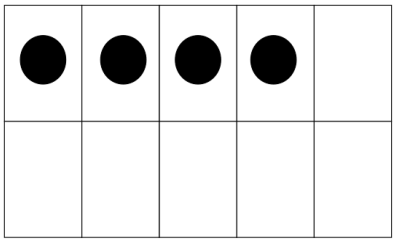


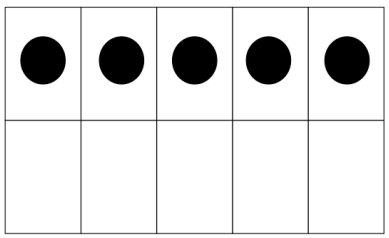


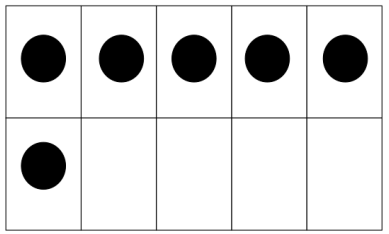


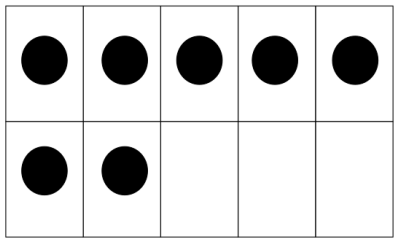


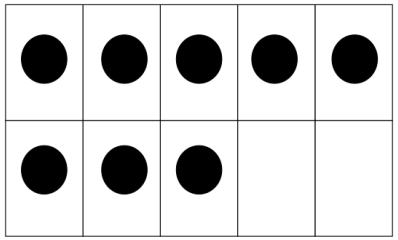


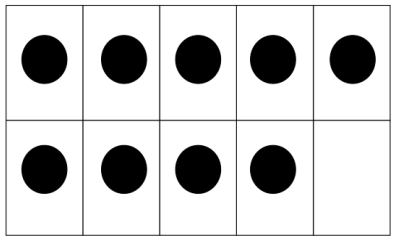


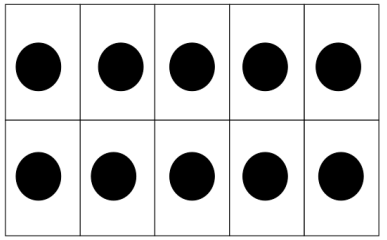


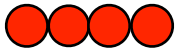


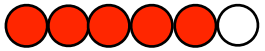


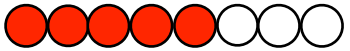




















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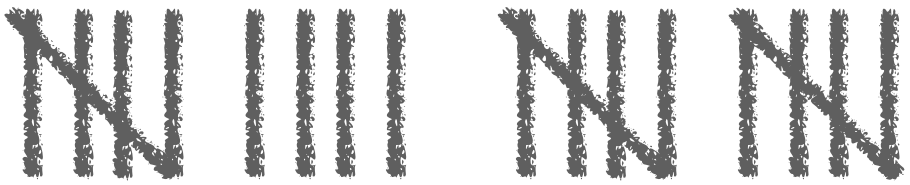

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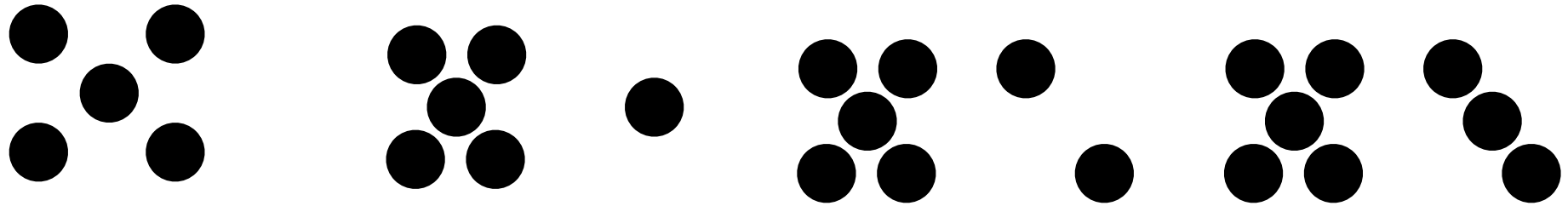
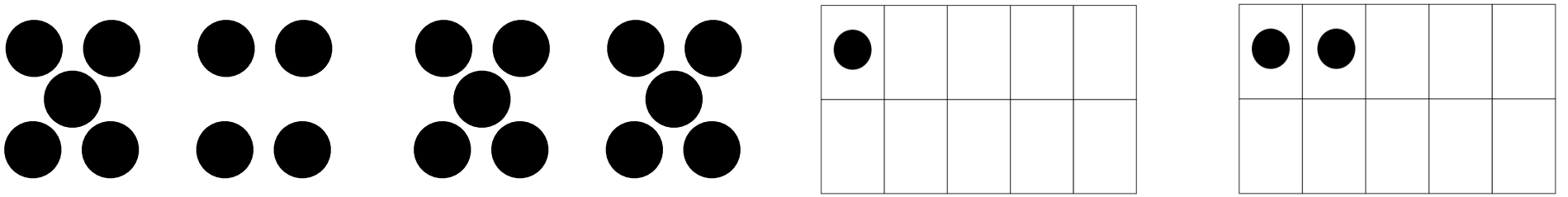
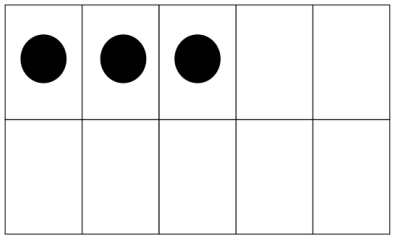
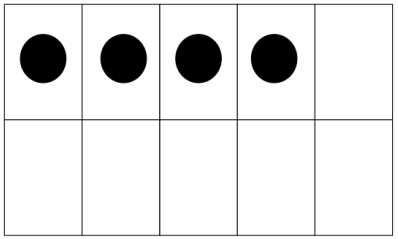
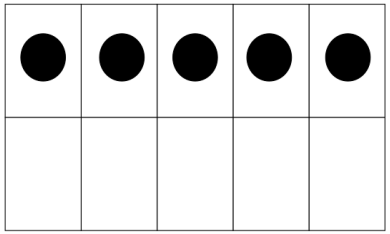
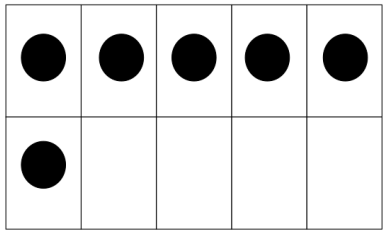


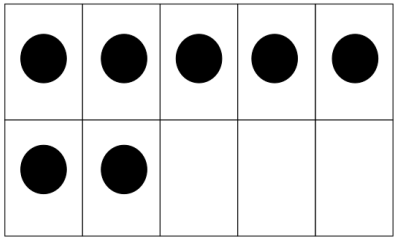


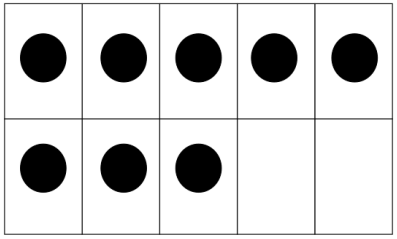


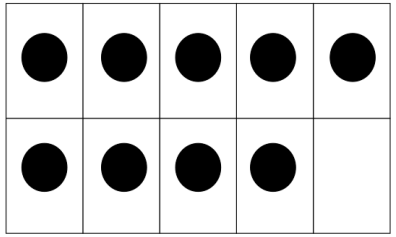


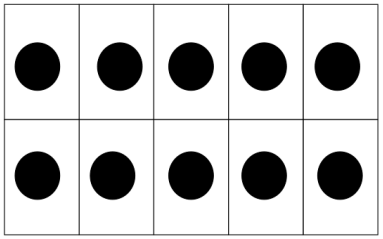




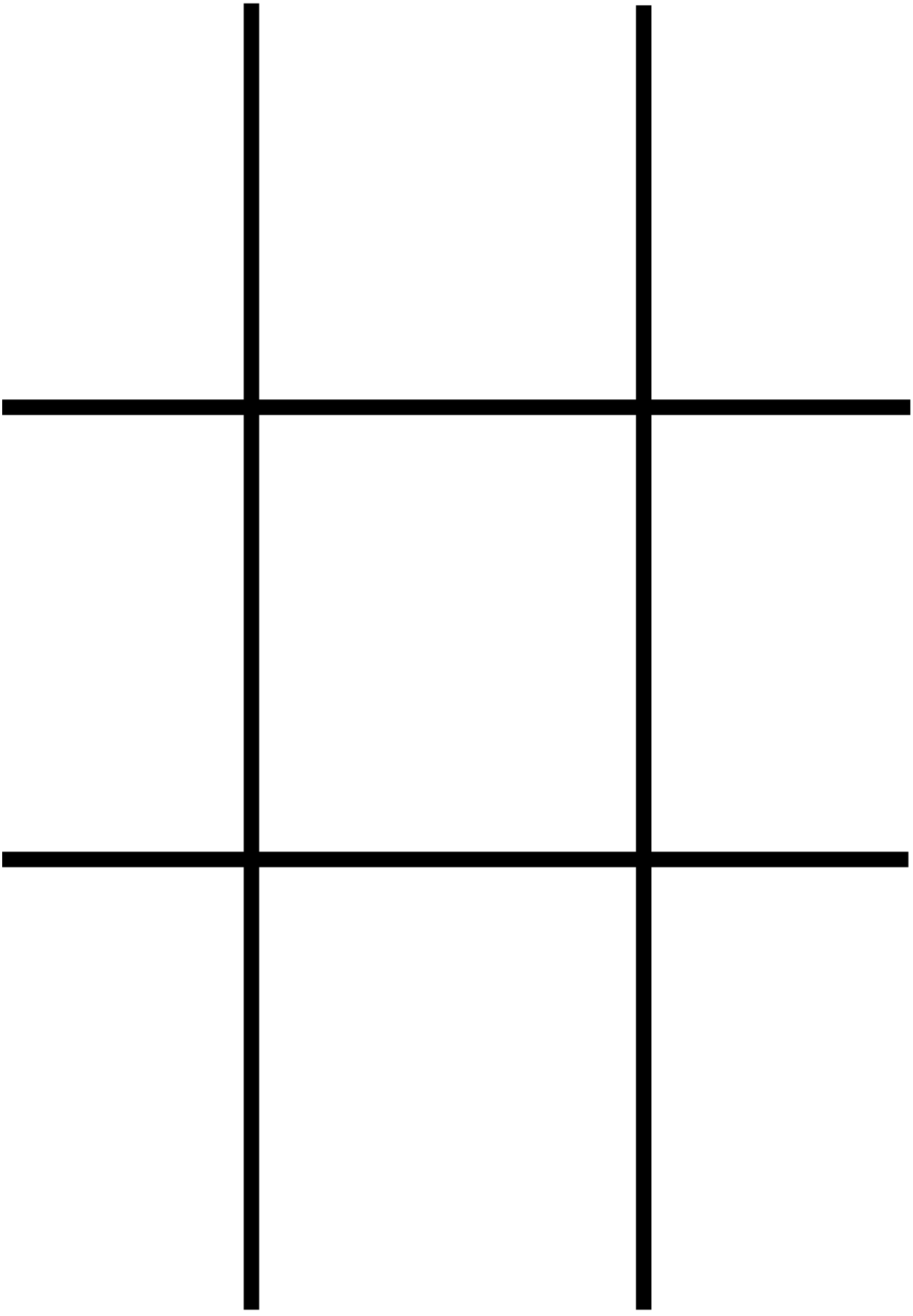








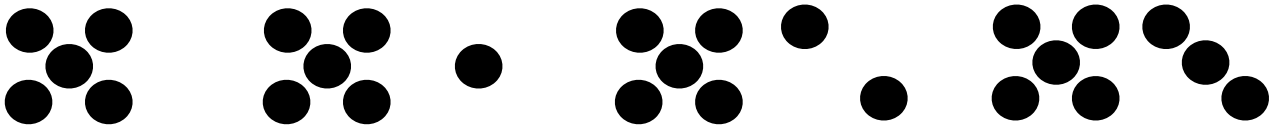
Blackline Master #4 - Tic-Tac-Toe



Blackline Master #5 - Dot Pattern Dominoes

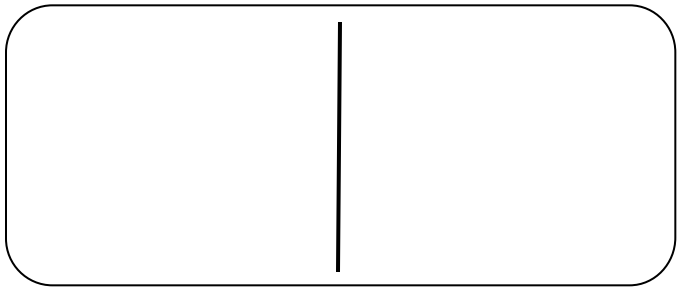
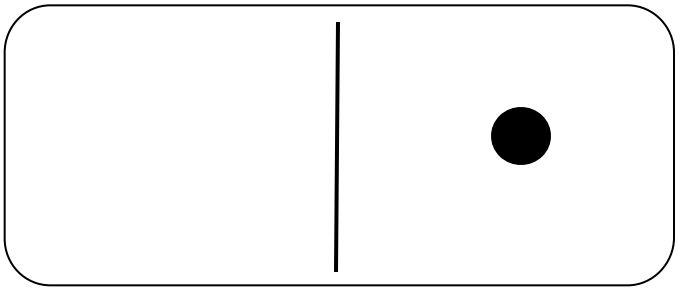
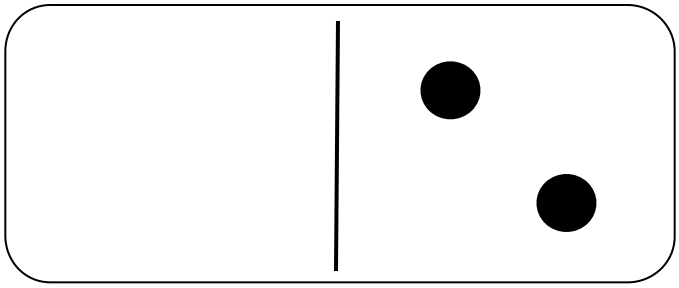
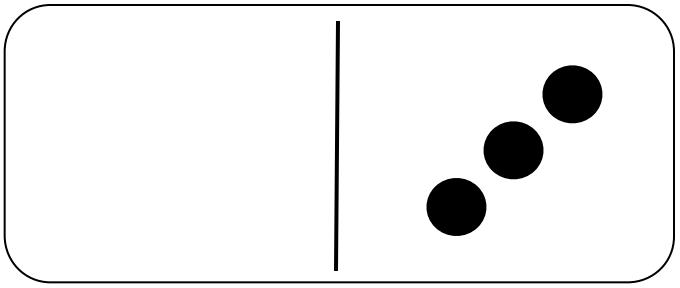
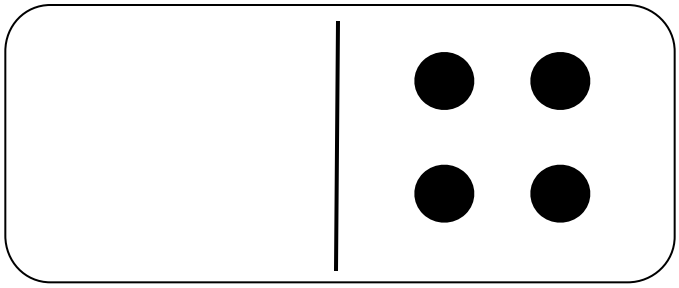
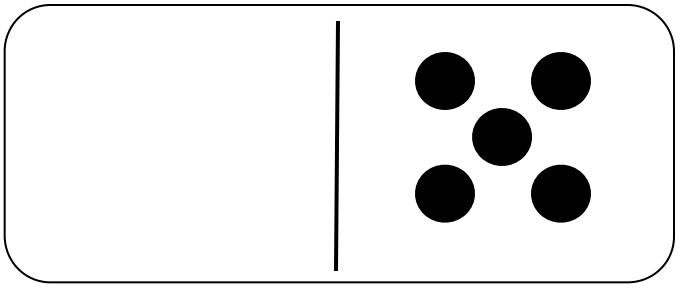
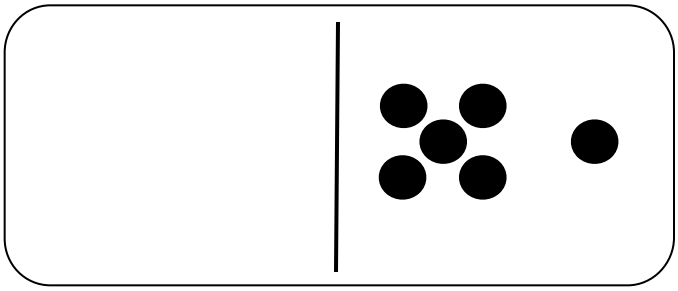
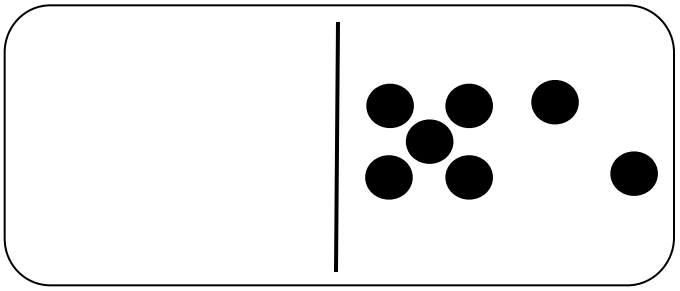
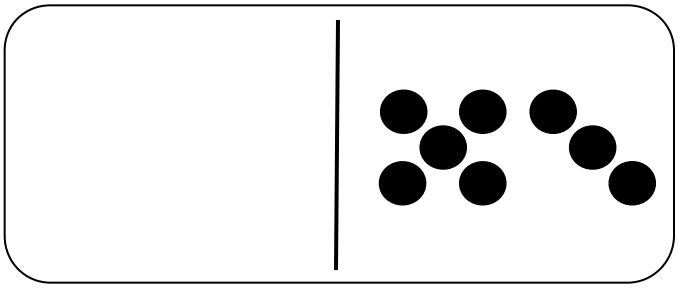
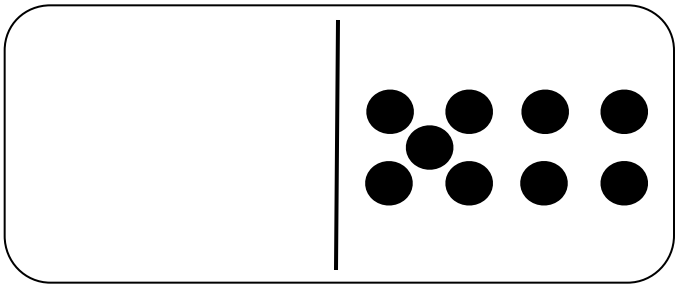
Dot Pattern Dominoes can help build connections between numbers for children. Too many children see 7 as just 7 things. They do not see that it is one more than six or two more than five. Most of the manipulatives we use in early elementary education do not help build connections between amounts. When using dot pattern dominoes, children quickly start to see the connections between numbers that are not readily apparent when using other dominoes (or other manipulatives). Here is what 5, 6, 7, and 8 look like in a typical domino set:

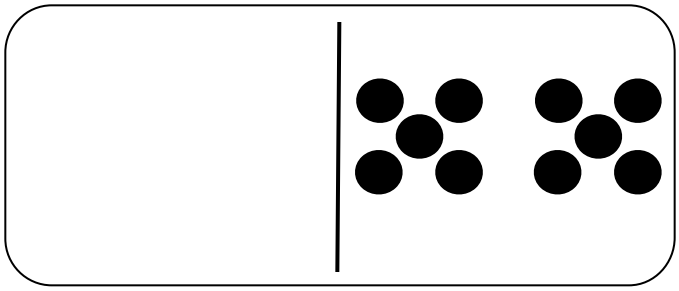
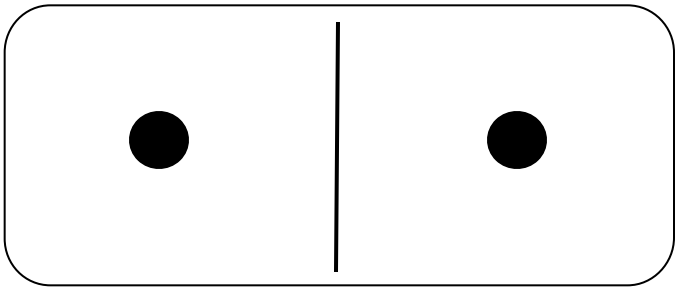
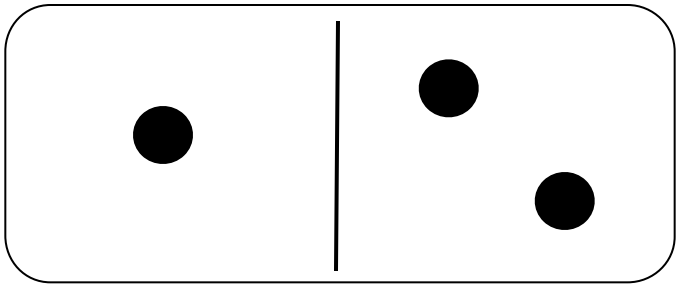
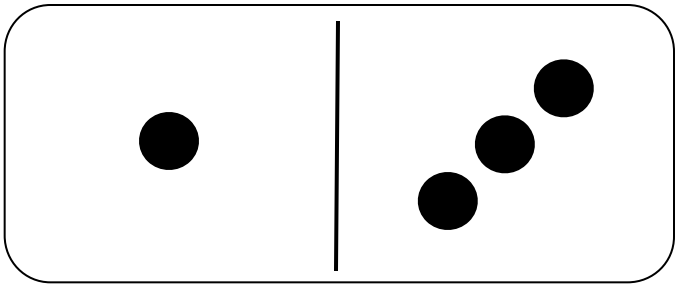
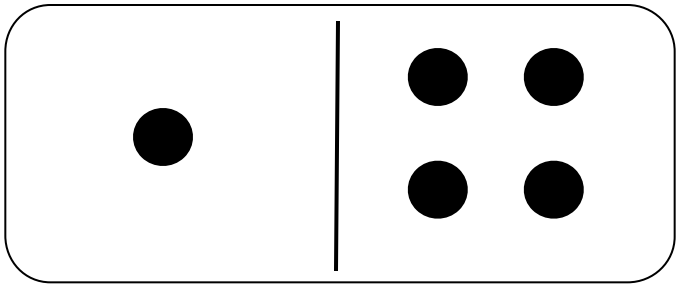
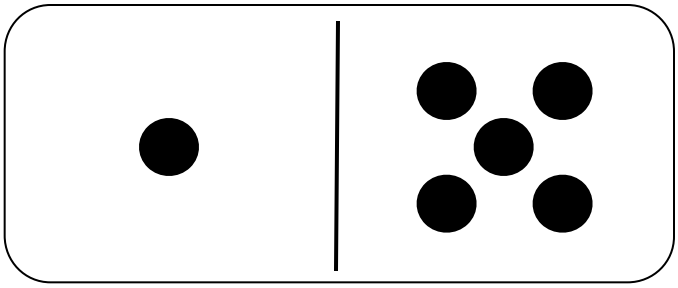
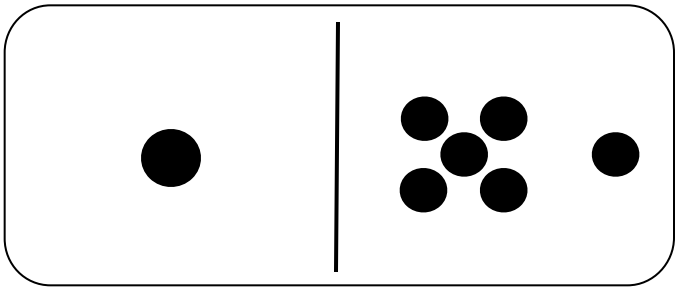
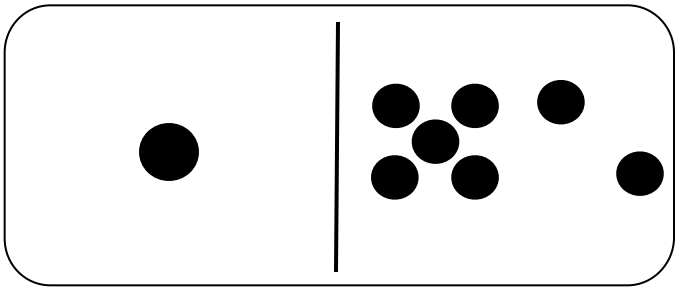
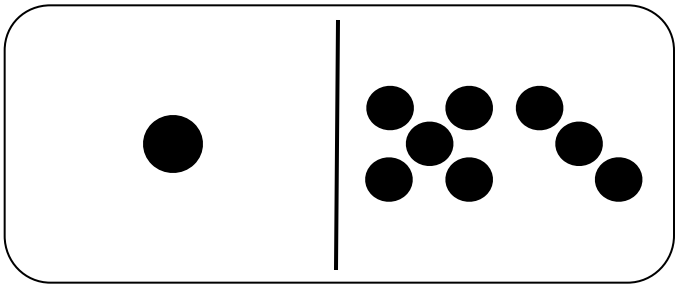
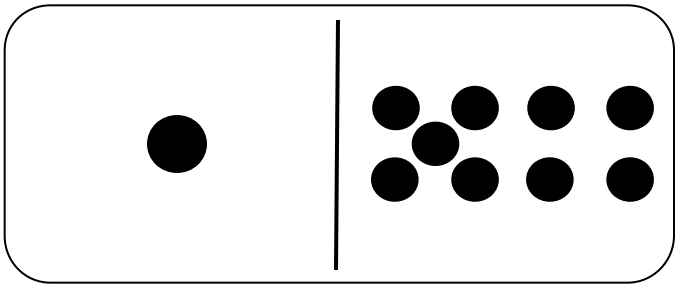
The first issue I have with this is that young children might instantly recognize, also known as subitize, the 5 and 6, but they end up counting to determine the amount on any domino pattern larger than the six. The other thing that is hard, especially for our struggling learners, is that they do not see how the five connects to the six, or how the six connects to the seven. Knowing those ideas is essential when we ask children to think about adding 6 + 7 and we want them to think of that as 6+6+1. Having a good visual representation of that in their minds helps them to manipulate the numbers on paper, but the visuals presented in traditional dominoes do not help children see those connections. So, I designed a set of Double Zero through Double Ten dominoes to help children see the connections between numbers and especially each number's connection to five. If children can "find the fives" within numbers, it makes them easier to decompose and use to make friendly amounts (that is why we do tally marks in groups of five). So take a look at how I represent 5, 6, 7, and 8 in these dominoes and think about how it might help children develop connections between the numbers and then how they will be able to use those connections to add sets:

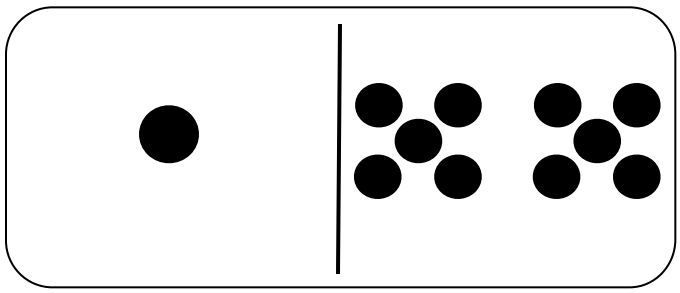
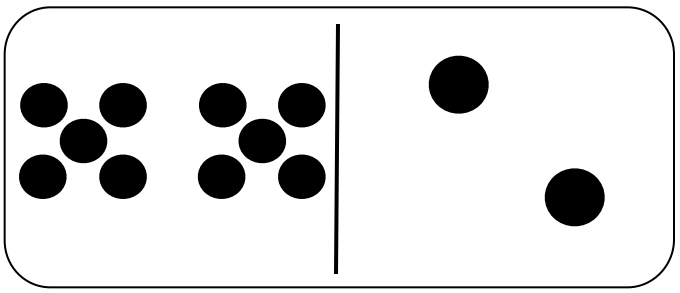
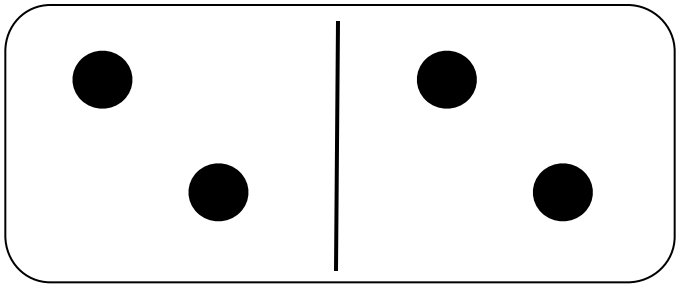
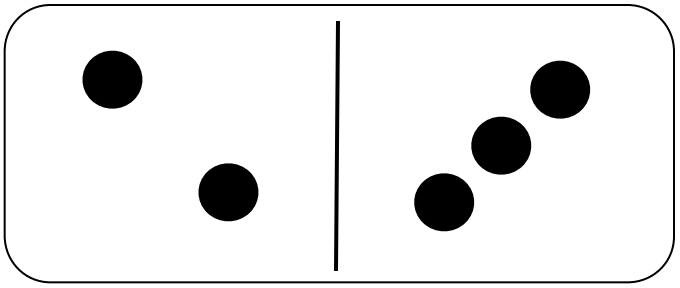
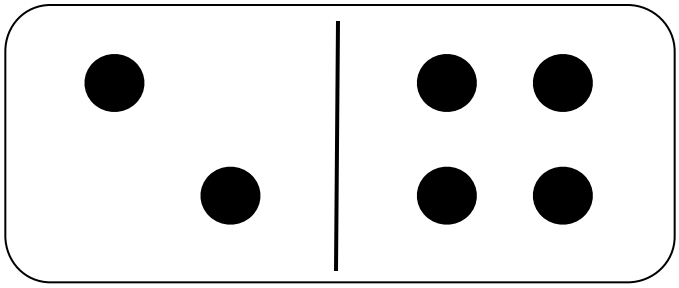
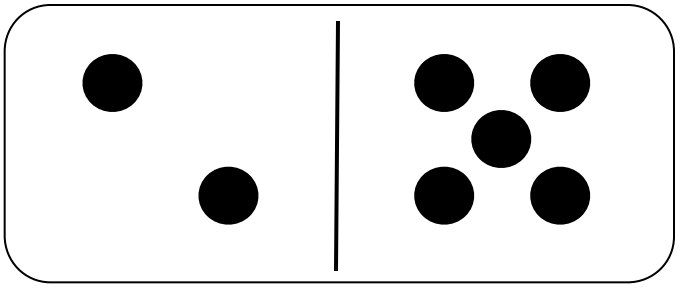
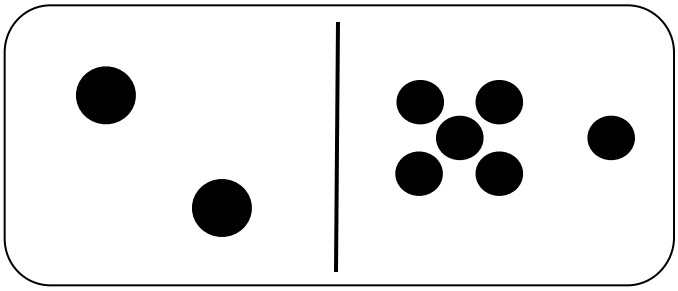
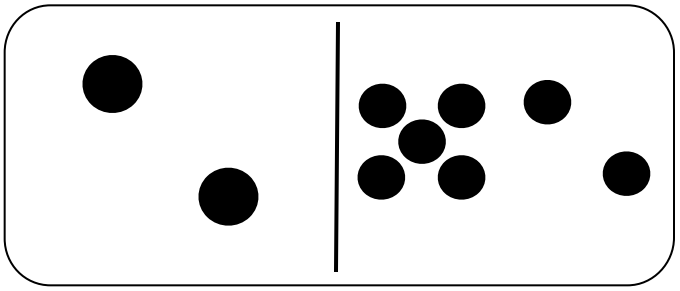
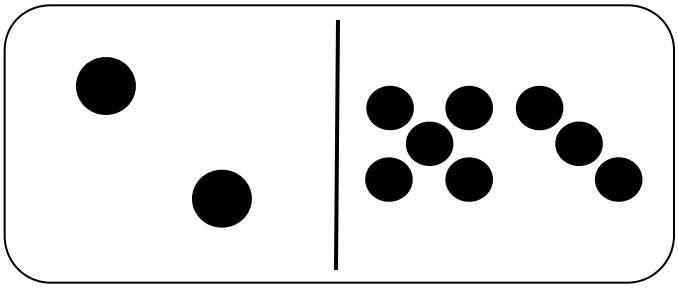
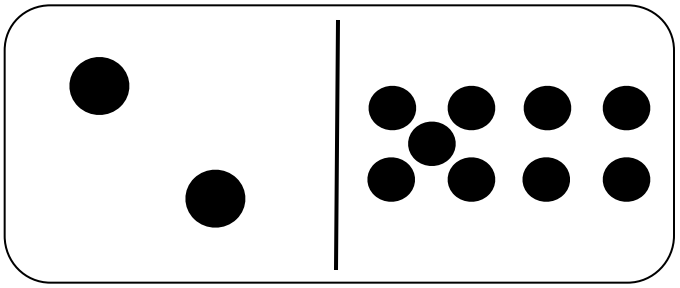


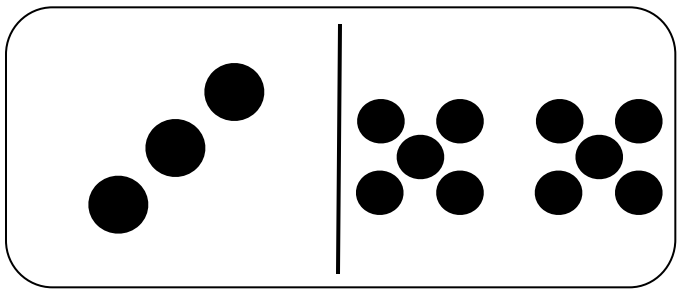
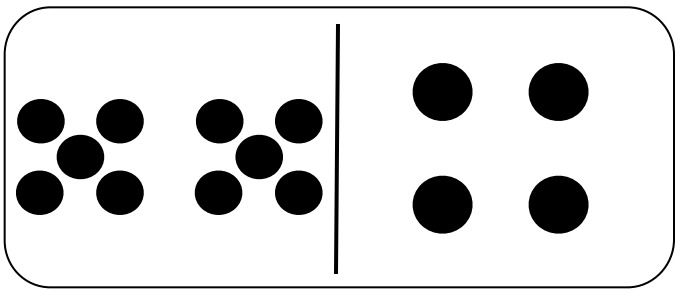
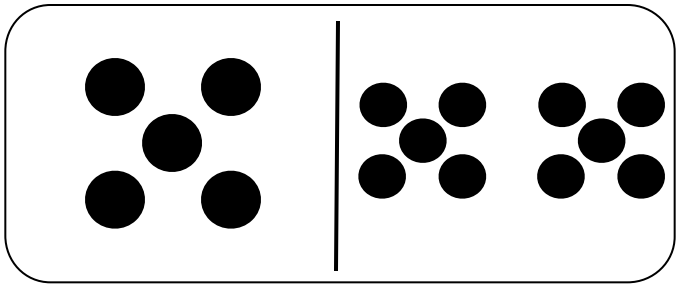
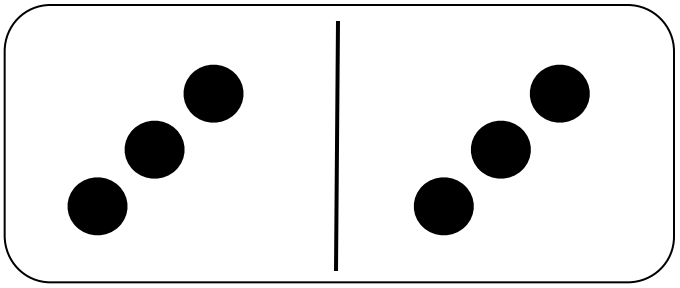
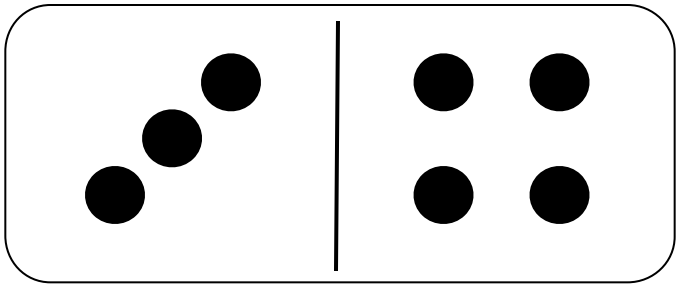
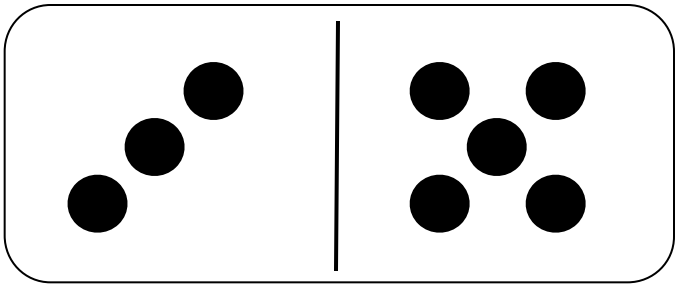
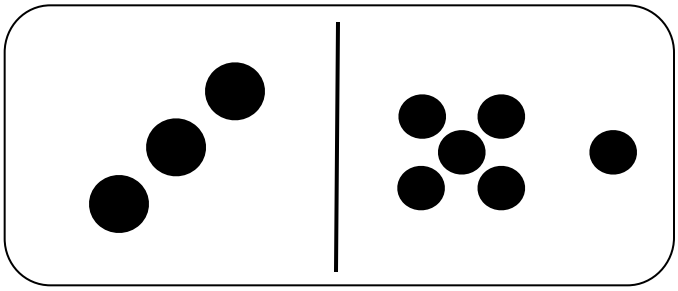
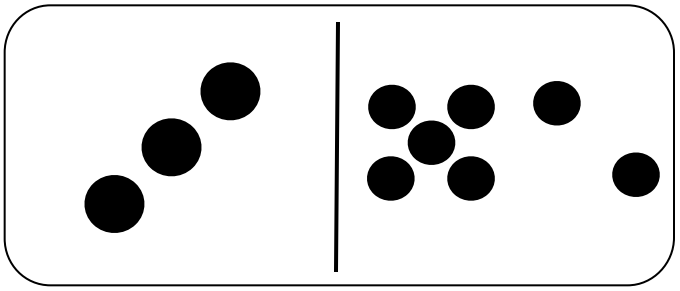
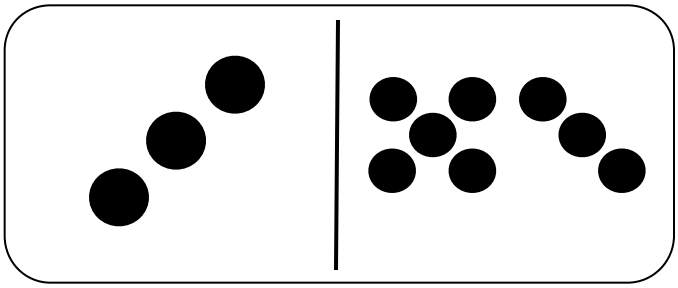
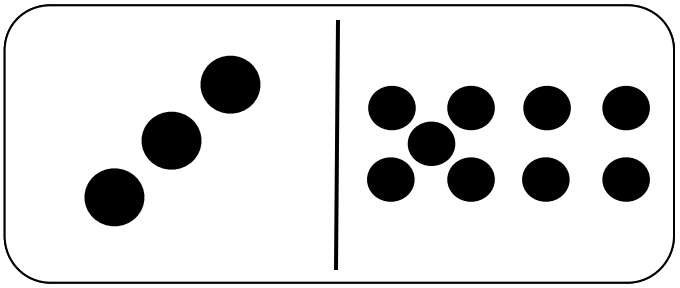
Ways to use these dominoes

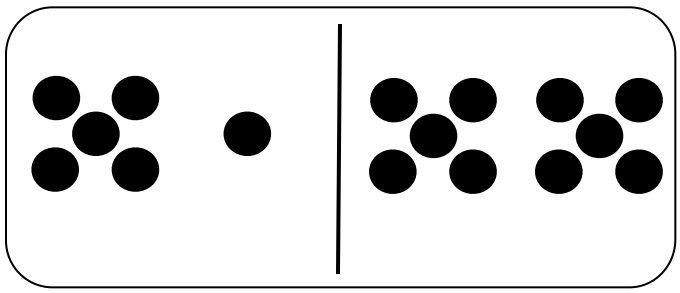
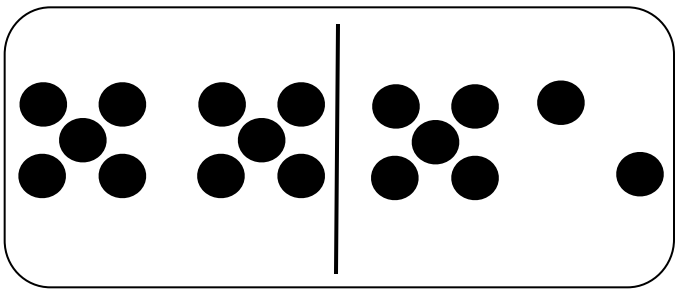
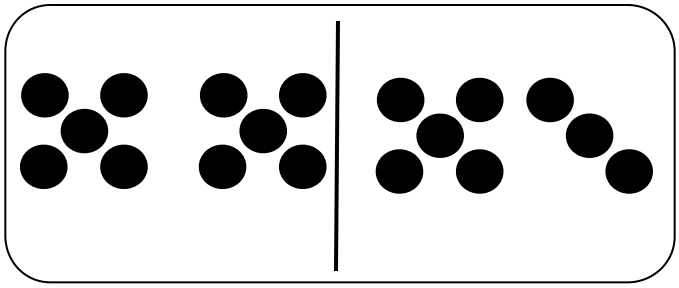
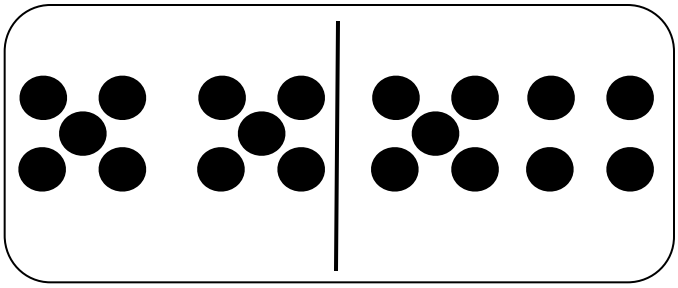
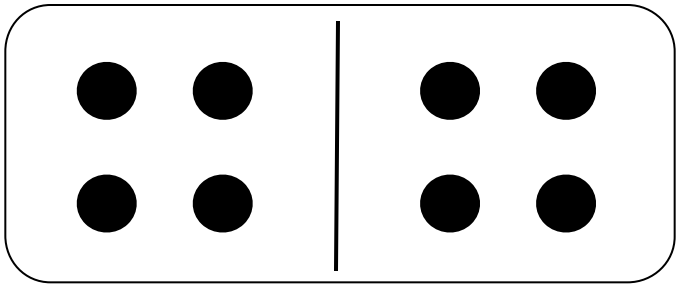
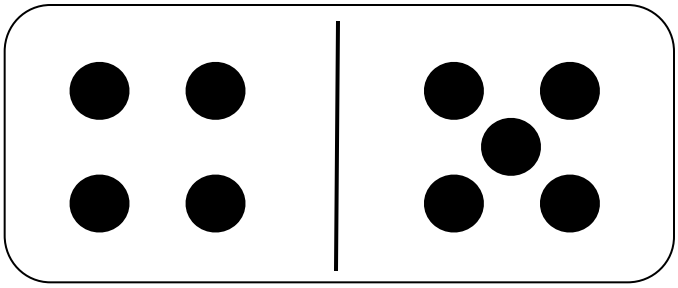
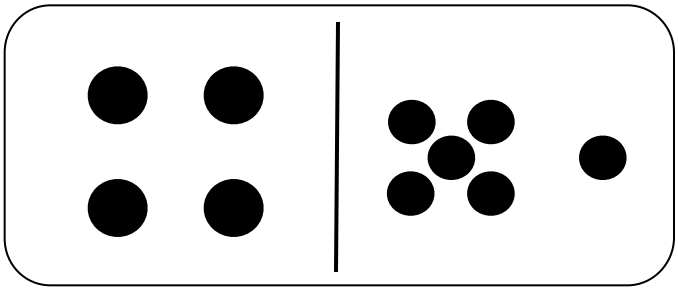
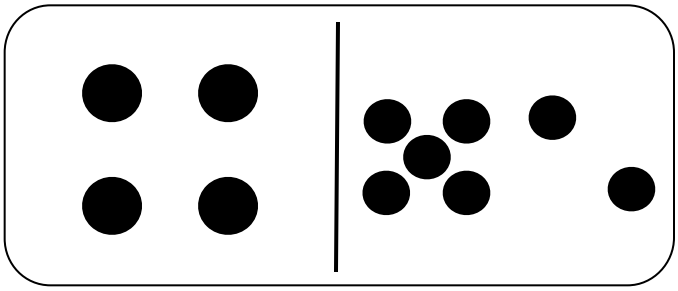
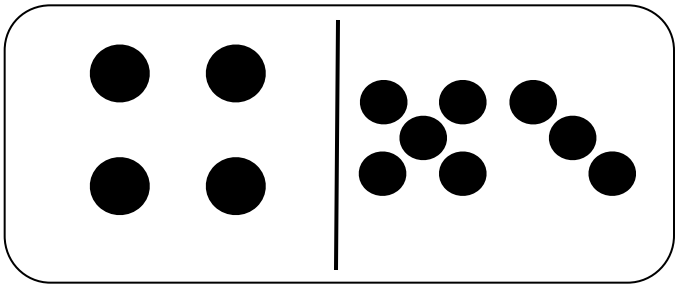
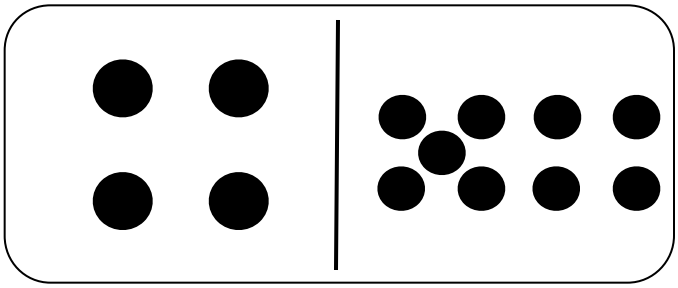
- Play the actual game of dominoes! ☺
- Domino War – divide the dominoes amount two players. Stack them face down in a pile, each player flips one over and the player with the larger total on the entire domino takes both dominoes.
- Use them as Quick Images – flash a domino for a few seconds and then hide it. Ask the students to tell you how many they saw. This encourages kids to see the groups and not count one-by-one.
- These are just a few; please feel free to email me on how you used them in your classroom! mathematicallyminded@yahoo.com

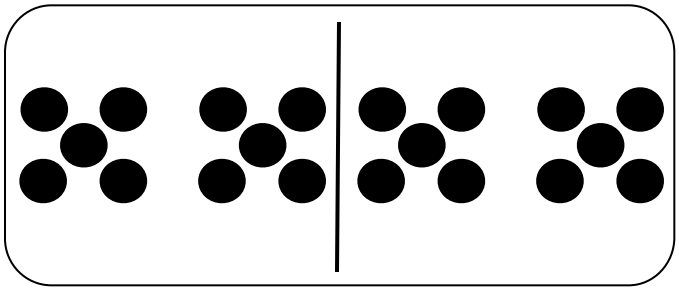
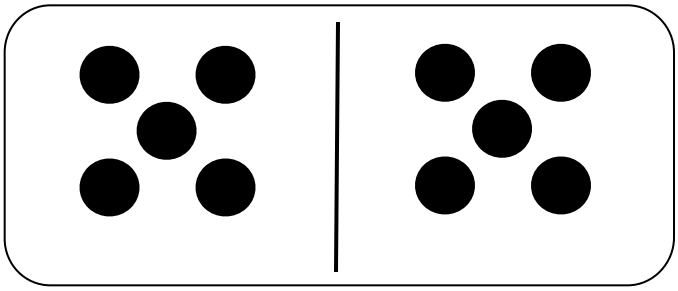
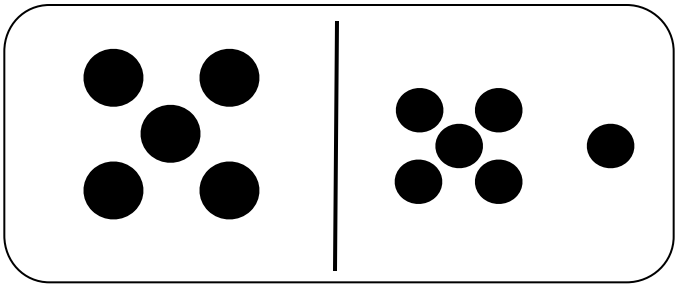
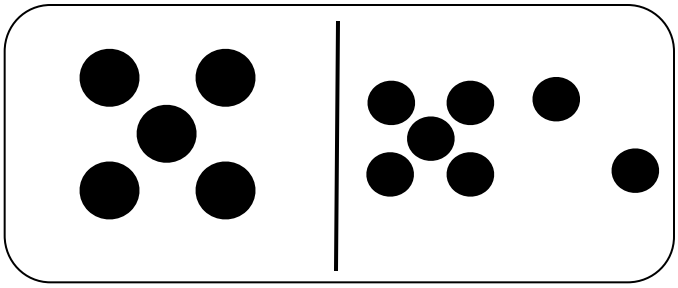
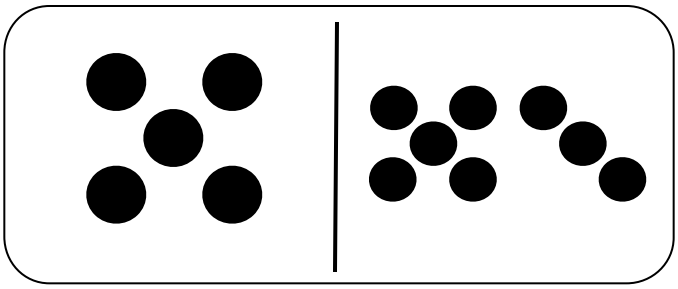
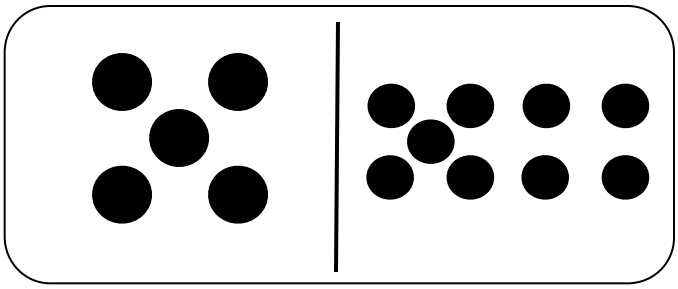
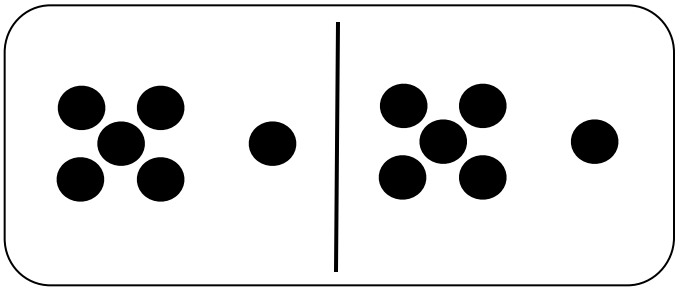
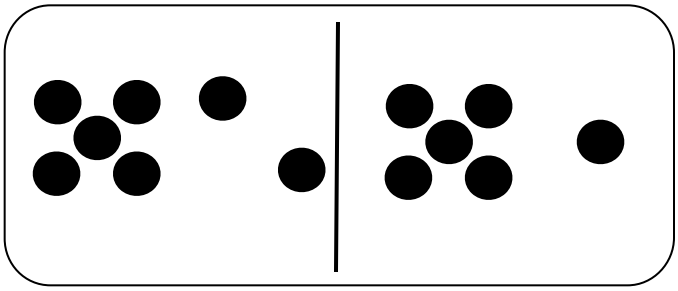
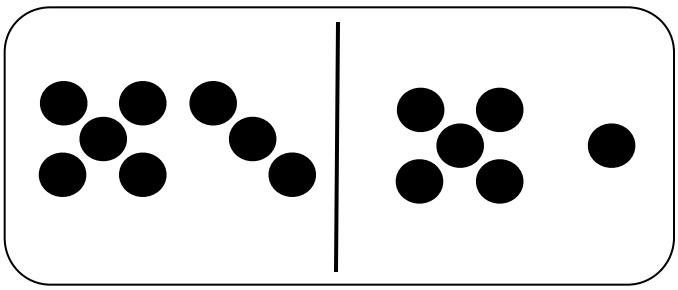
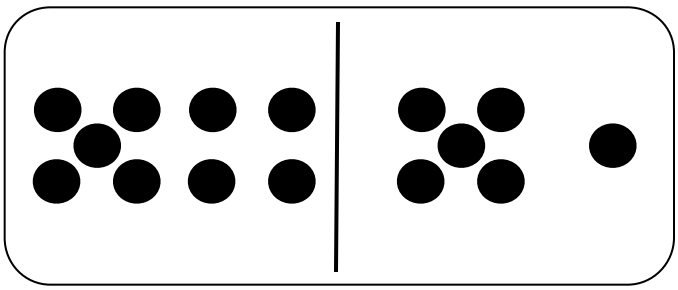











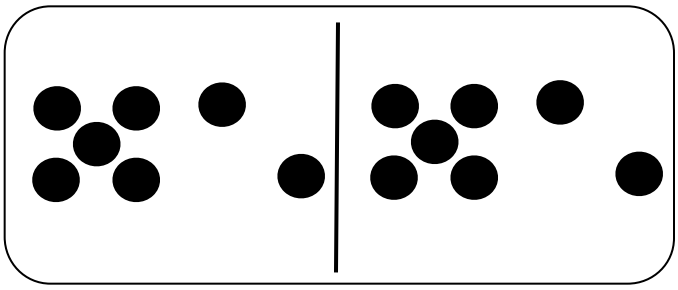











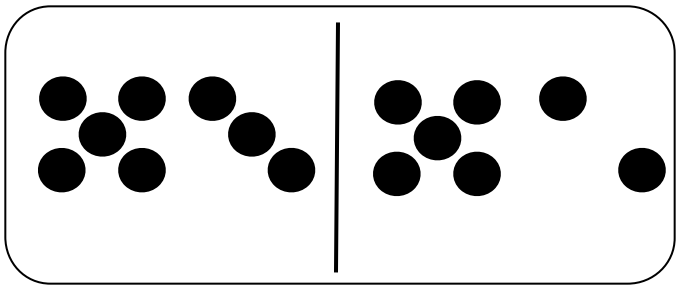











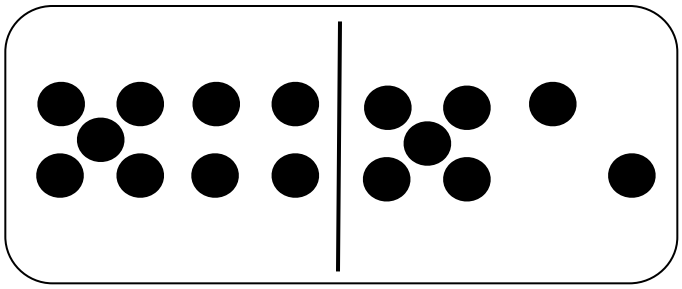











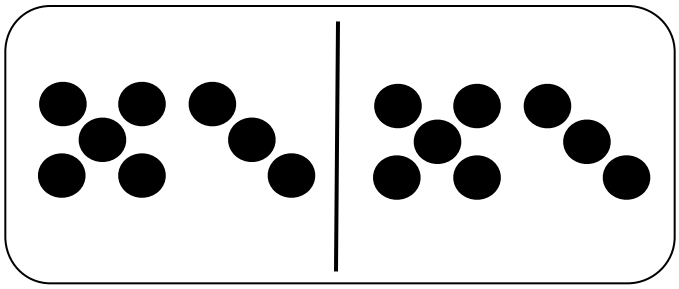











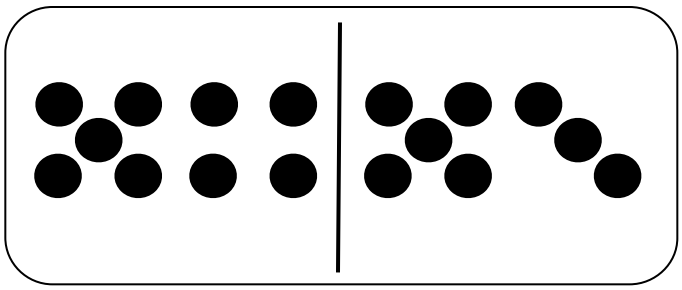











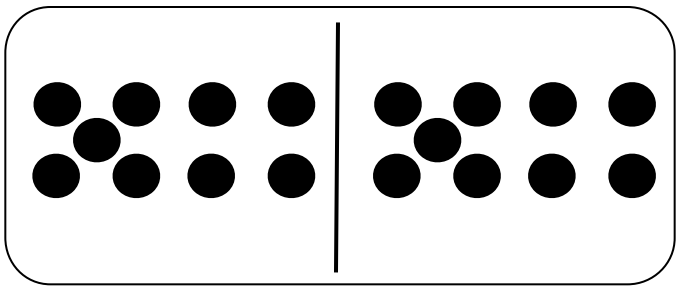






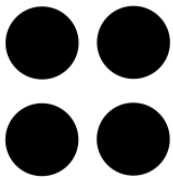
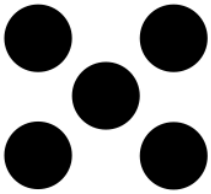
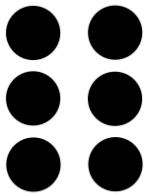
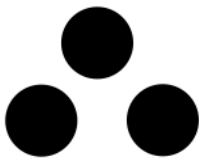
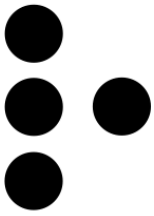
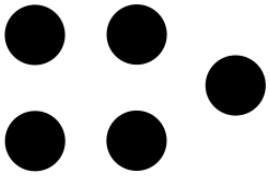




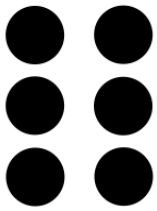
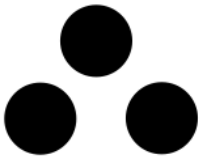
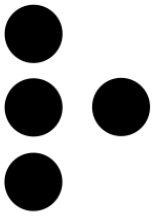
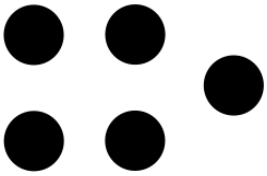
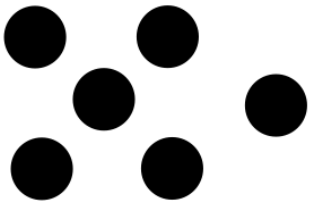
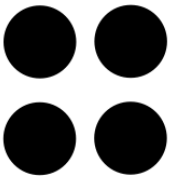
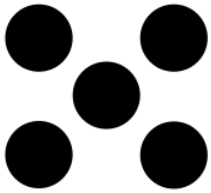


Blackline Master #6 - BINGO Boards

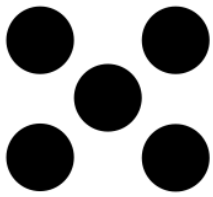
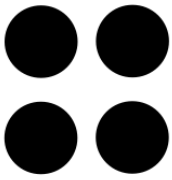
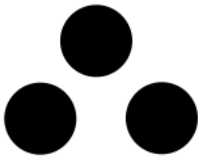
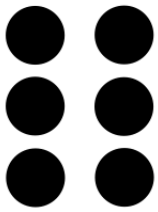
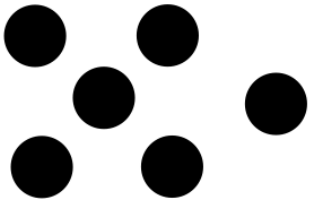
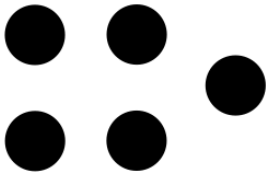
To cut out the game boards below, keep the board that has the dot patterns intact. The boards with the numbers need to be cut up so that children have them in pieces to cover the spots on the Dot Pattern BINGO Game Board. You may want to number the dot pattern game board along with the numeral game pieces and then put them in their own Ziploc bag, because each game board does not contain the same numerals. The boards at the end of the document contain more of the larger numbers, and can be given to children who are able to subitize the small amounts within the larger set.

1	2	3
4	5	6
3	4	5
		
		
		

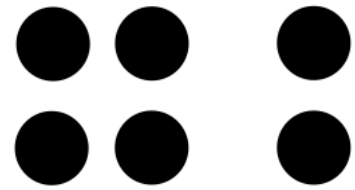
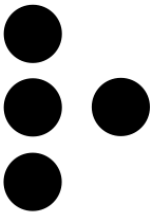
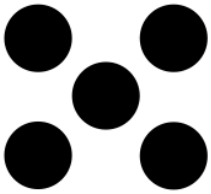
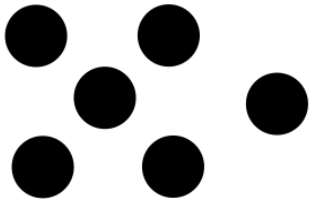
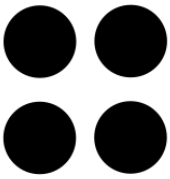
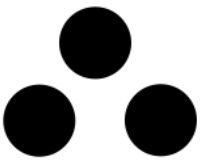
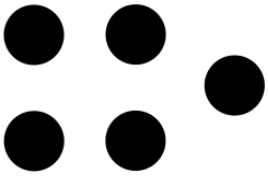
6	2	3
4	5	6
3	4	5

5	2	3
4	3	6
6	1	5

6	2	4
3	5	6
4	3	5

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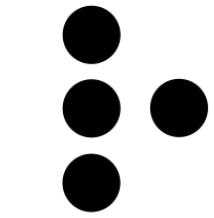		
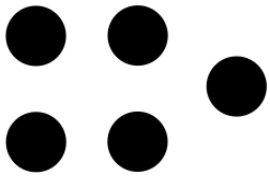	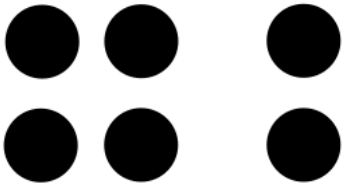	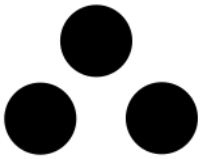
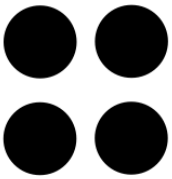		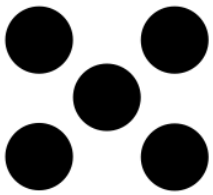

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		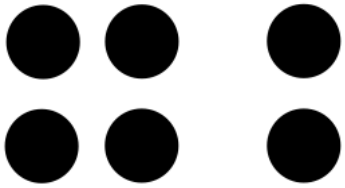
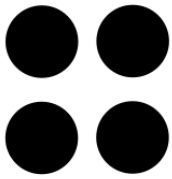		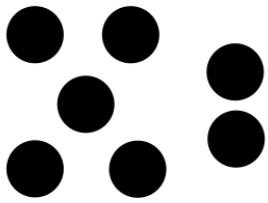
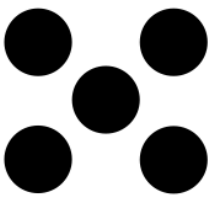	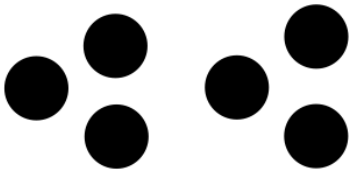	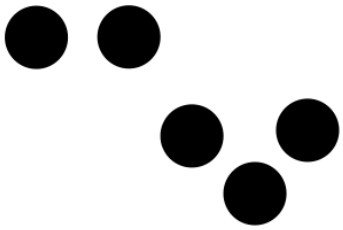

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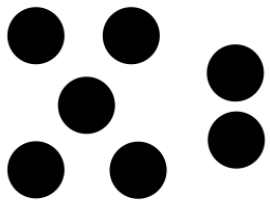		
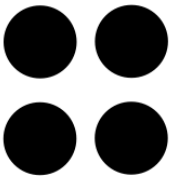	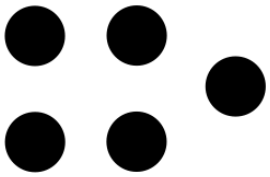	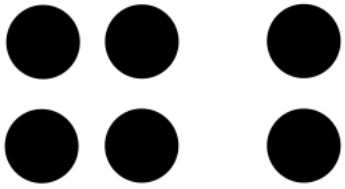
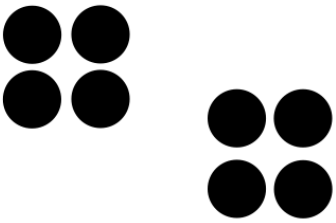	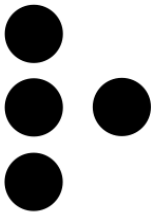	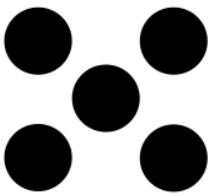

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	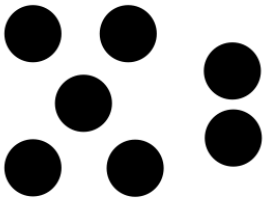	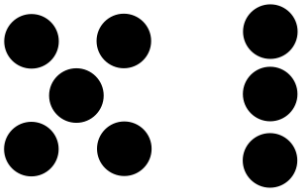
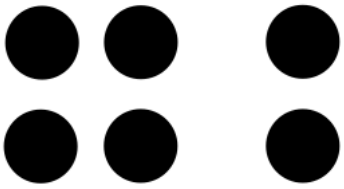	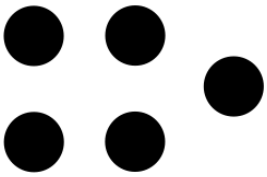	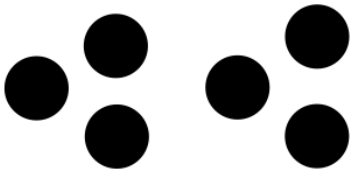
	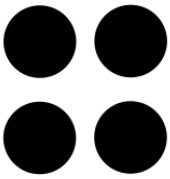	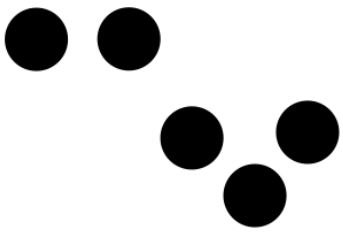

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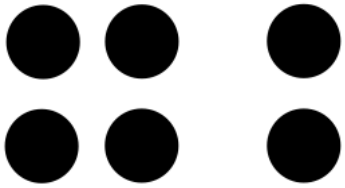	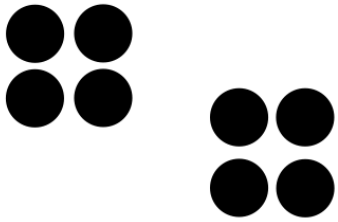	
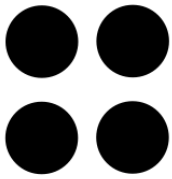	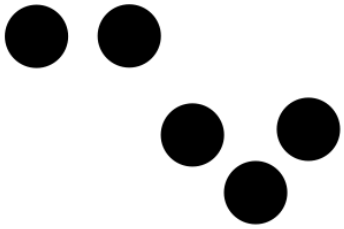	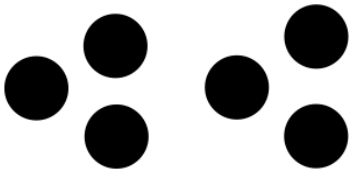
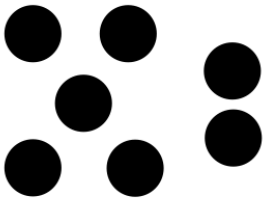	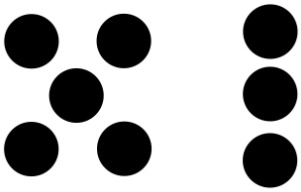	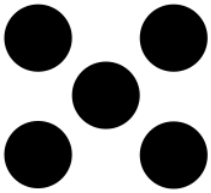

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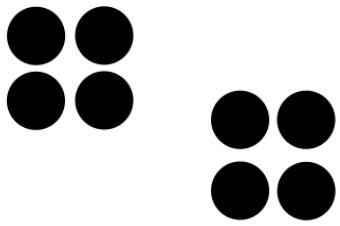	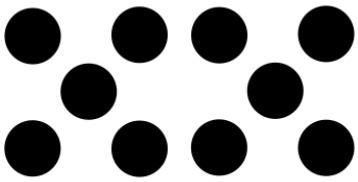	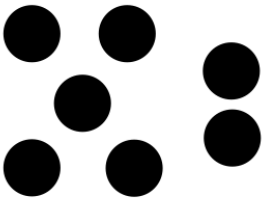
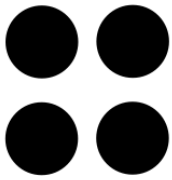	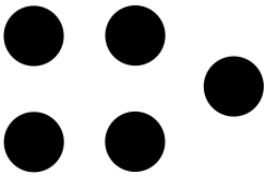	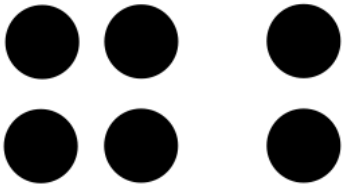
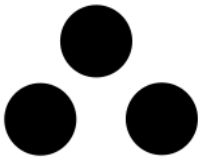	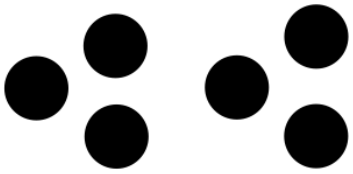	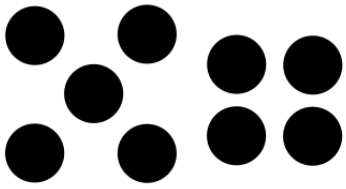

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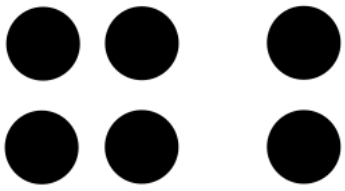	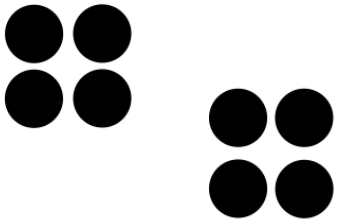	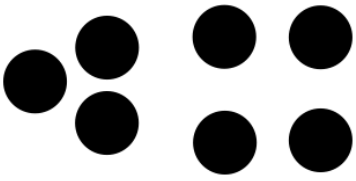
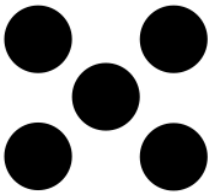	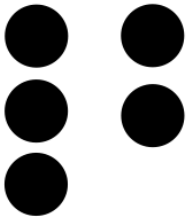	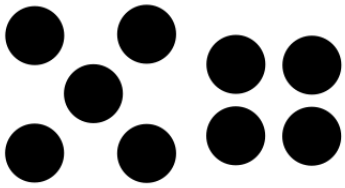
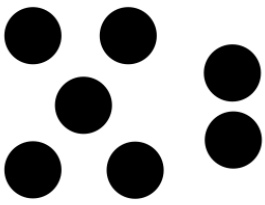	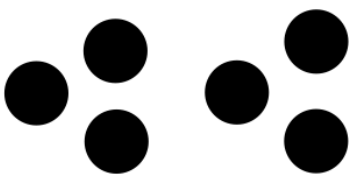	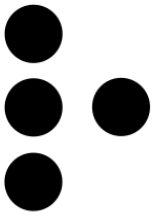

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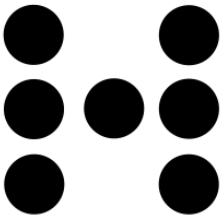	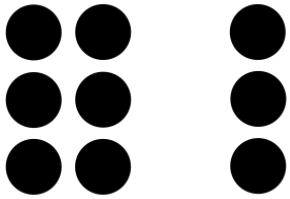	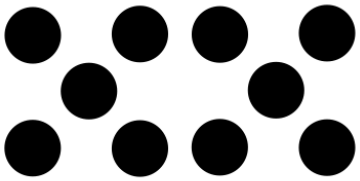
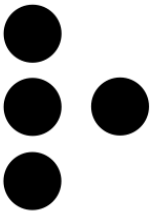	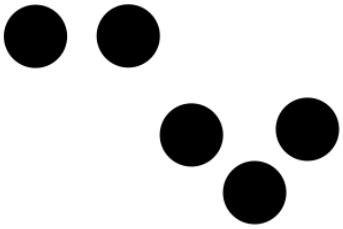	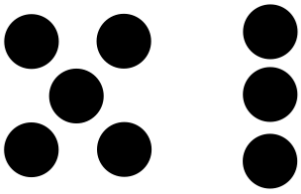
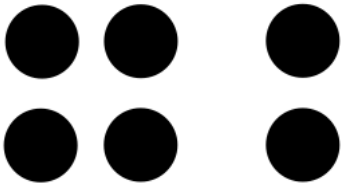	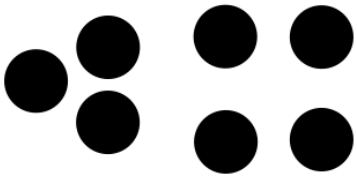	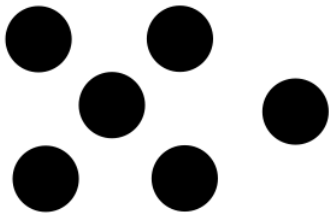

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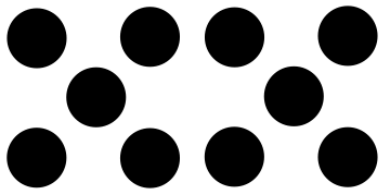	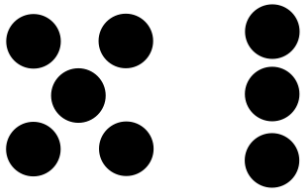	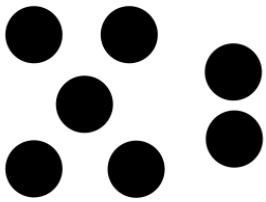
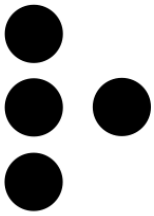	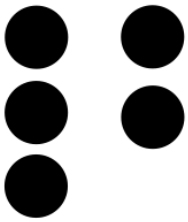	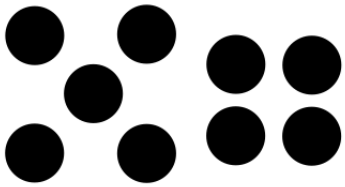
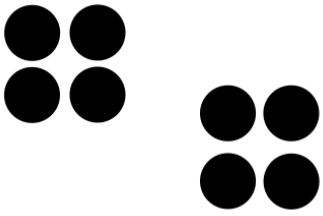	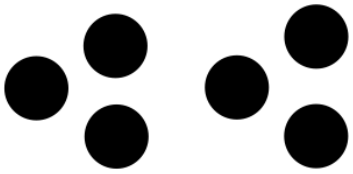	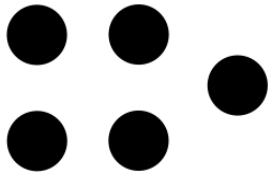

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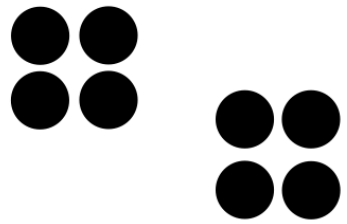	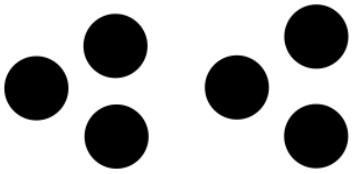	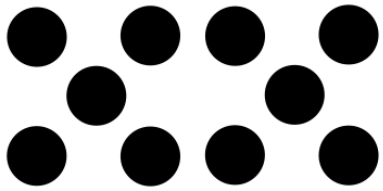
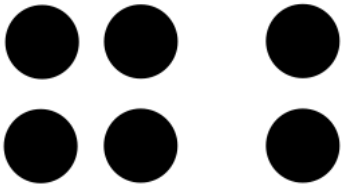	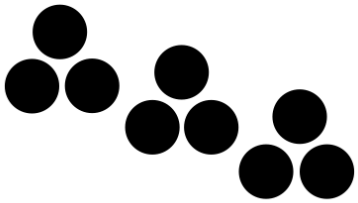	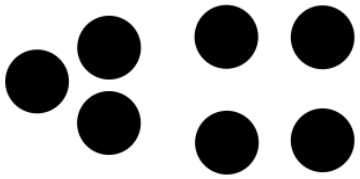
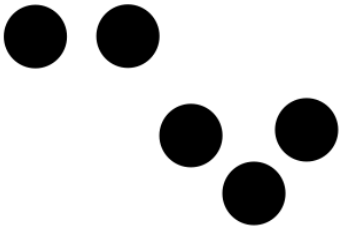	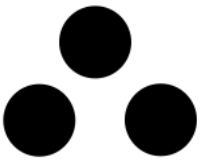	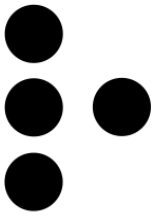

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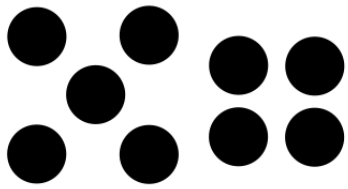	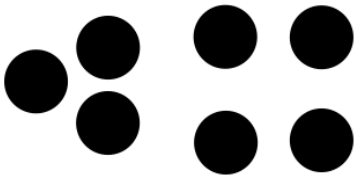	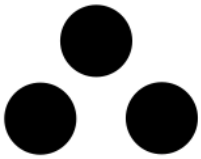
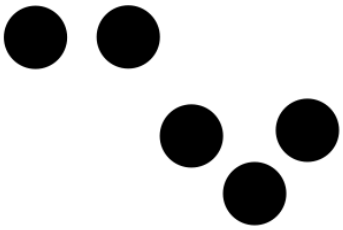	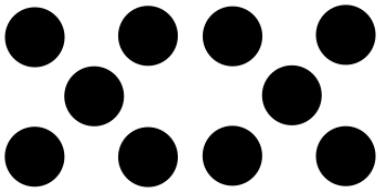	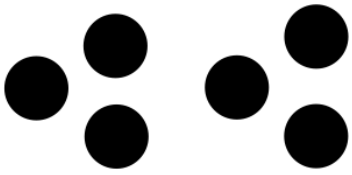
	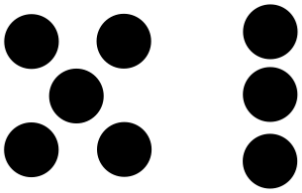	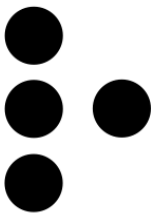

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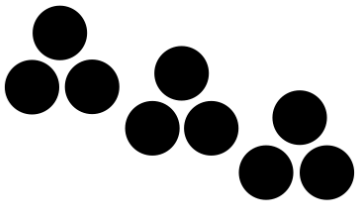	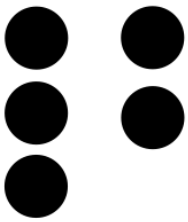	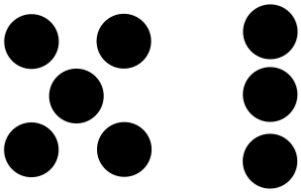
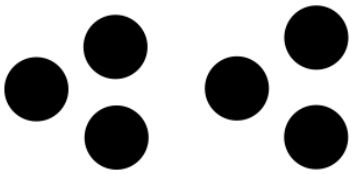	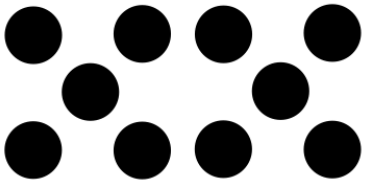	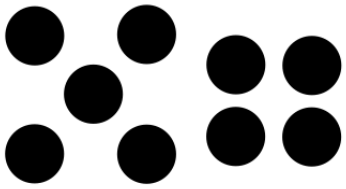
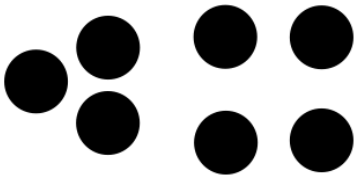	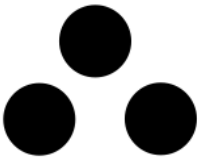	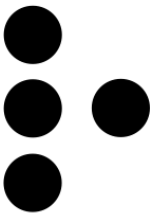

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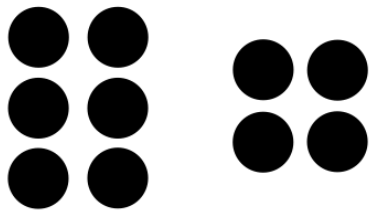	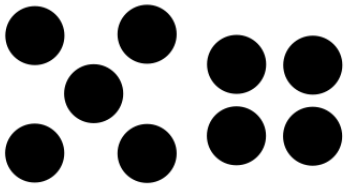	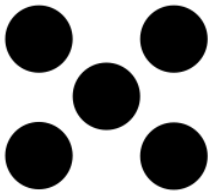
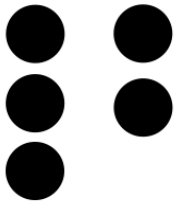	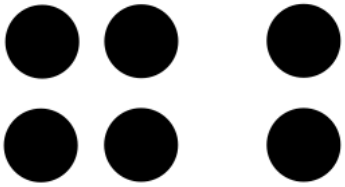	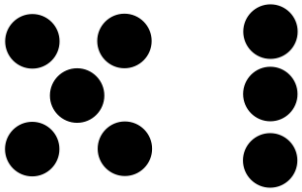
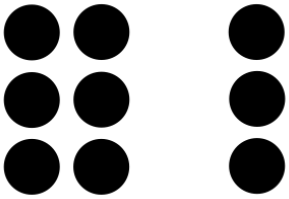	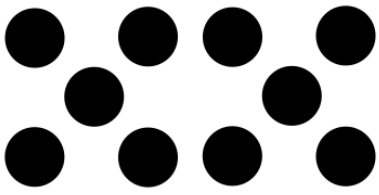	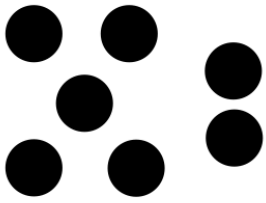

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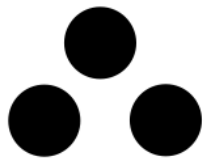	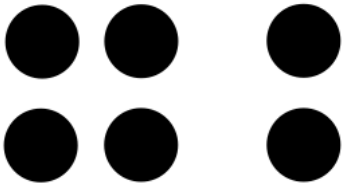	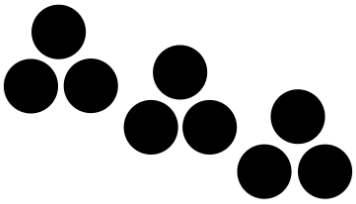
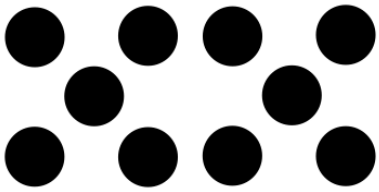	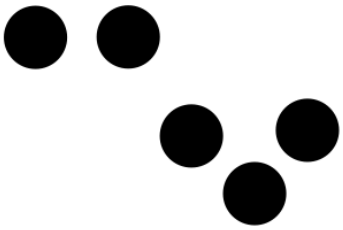	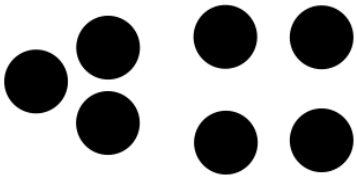
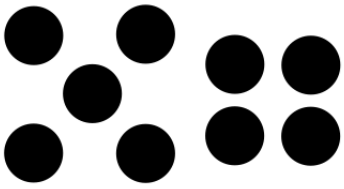	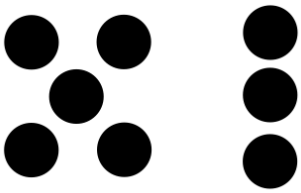	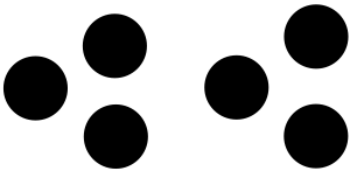

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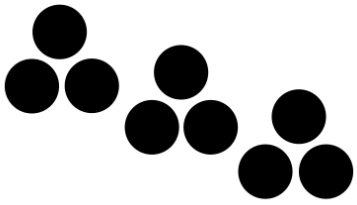
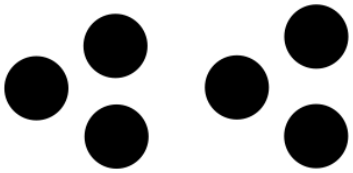
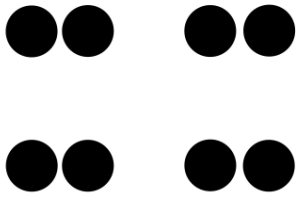
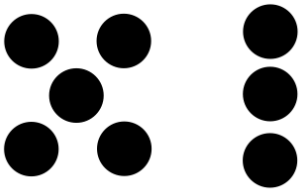
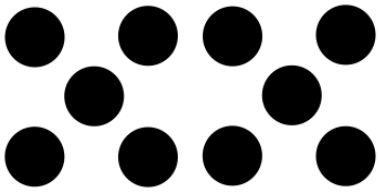
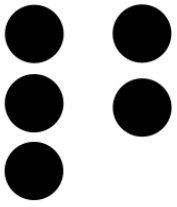
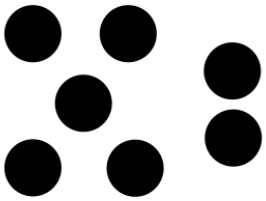
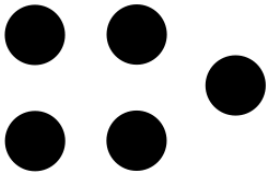
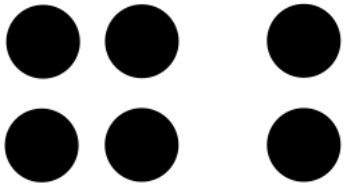
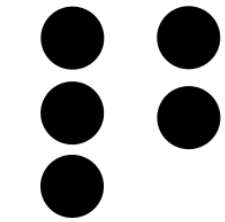
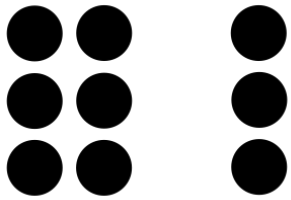
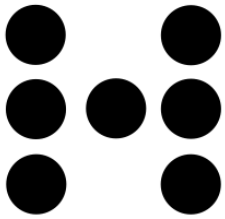
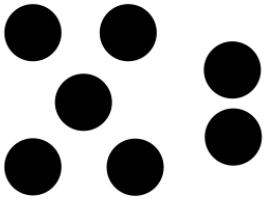
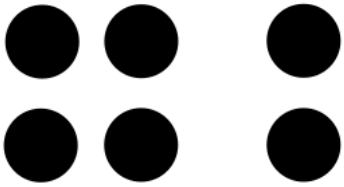
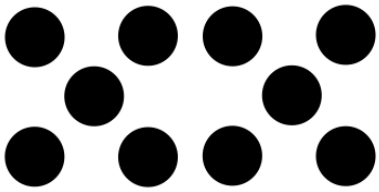
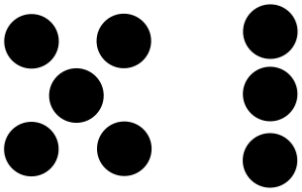
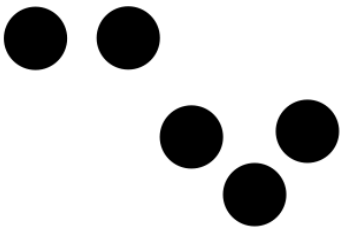
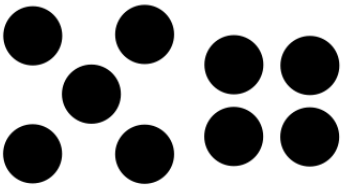
7	5	6

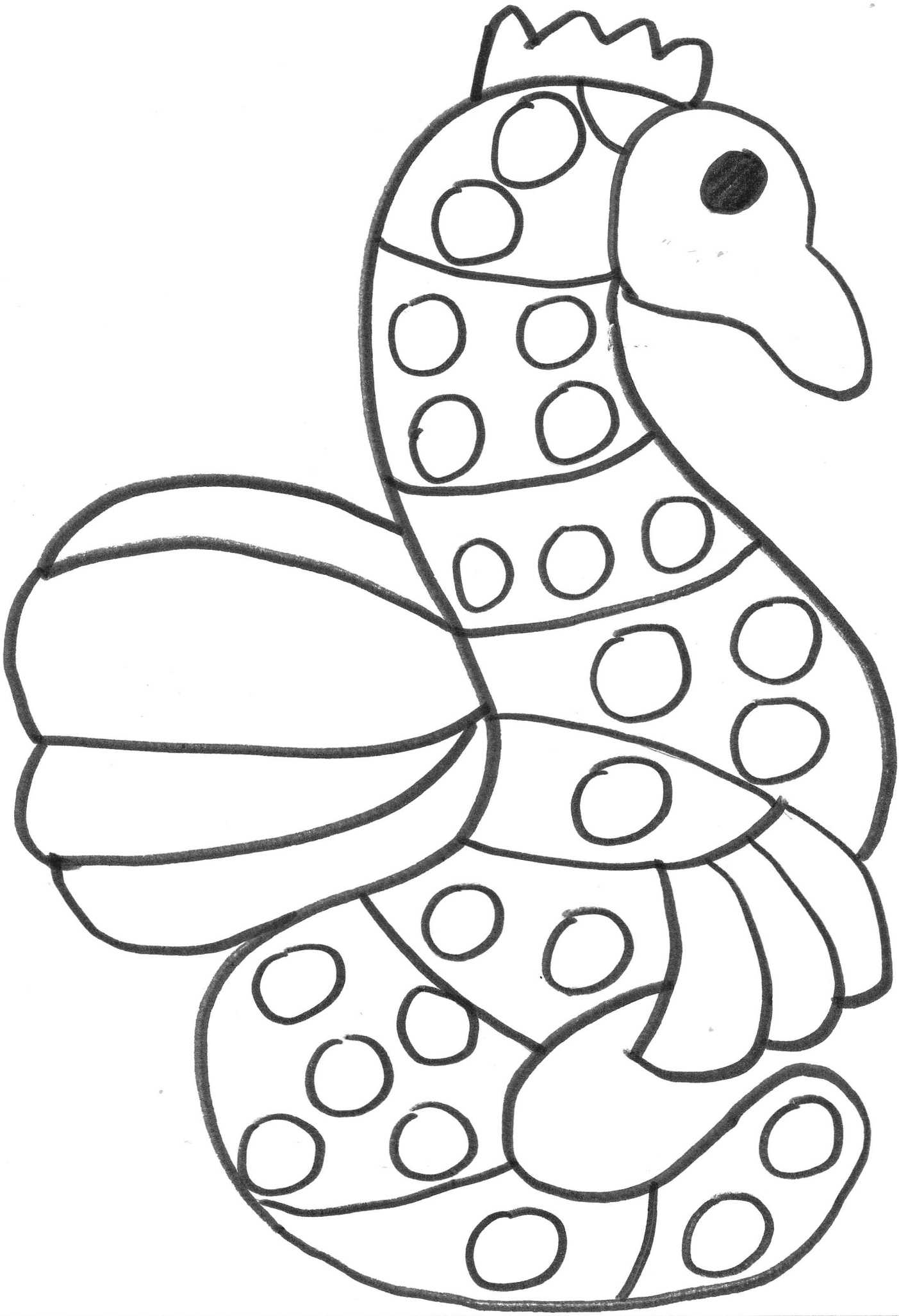
		
		
		

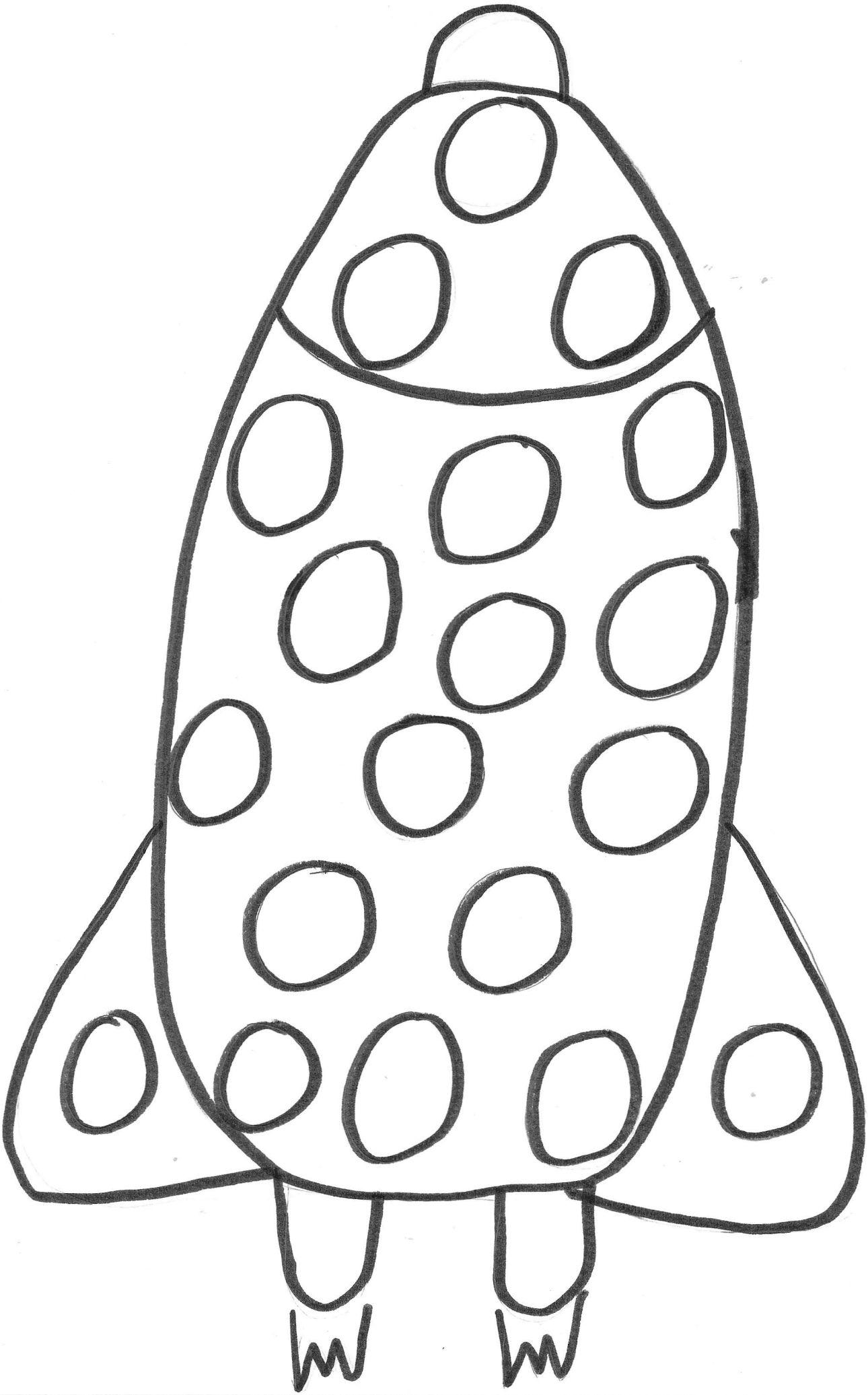
5	9	7
7	6	10
8	5	9

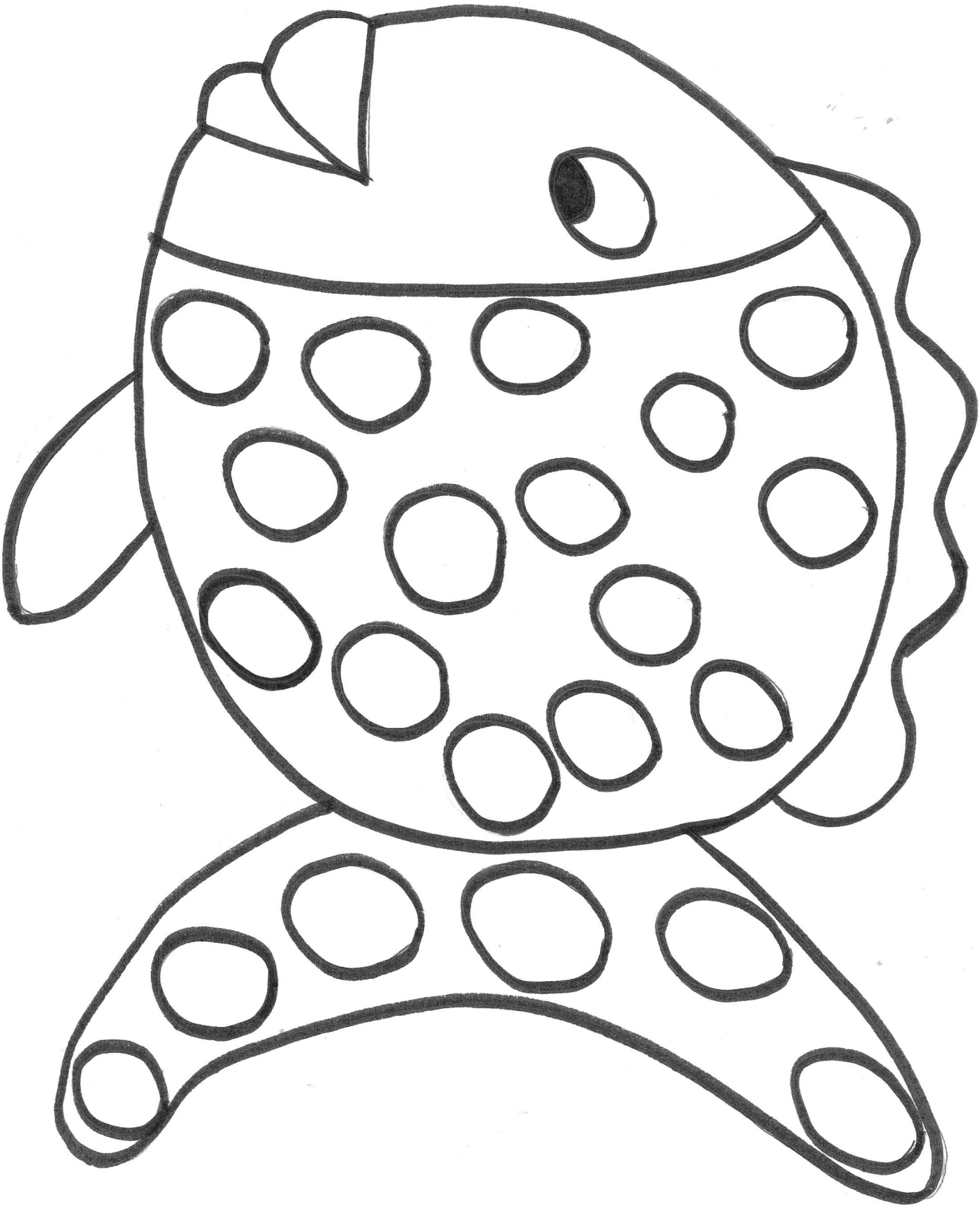
		
		
		

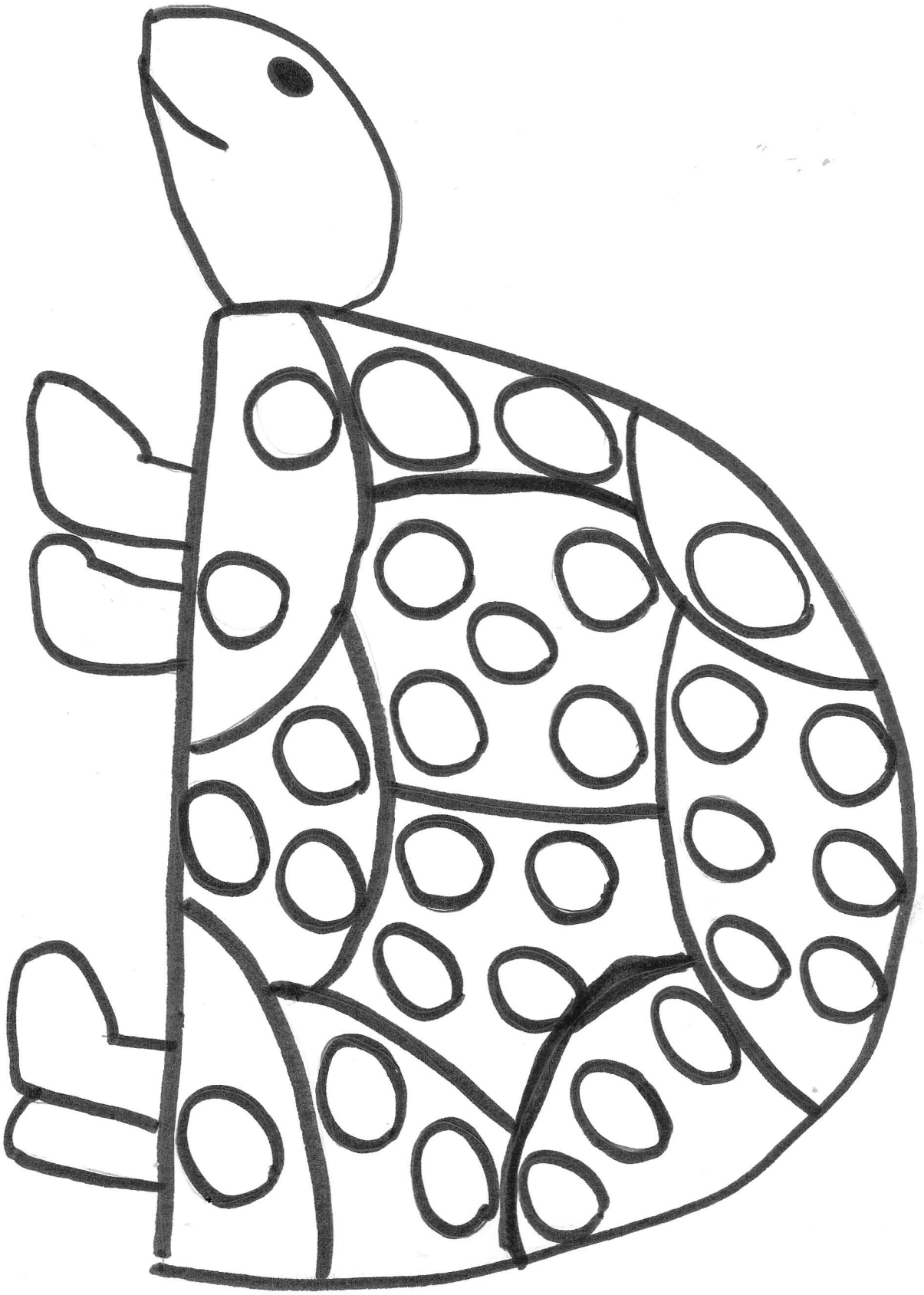
Blackline Master #7: Subitizing Mats

I found this idea on the site Learning 4 Kids; see the post about the activity here http://www.learning4kids.net/2012/10/28/counting-subitising/. I drew these pictures freehand by looking at the pictures on her site. She now has some templates on her site, so if you want more pictures you can find more there.







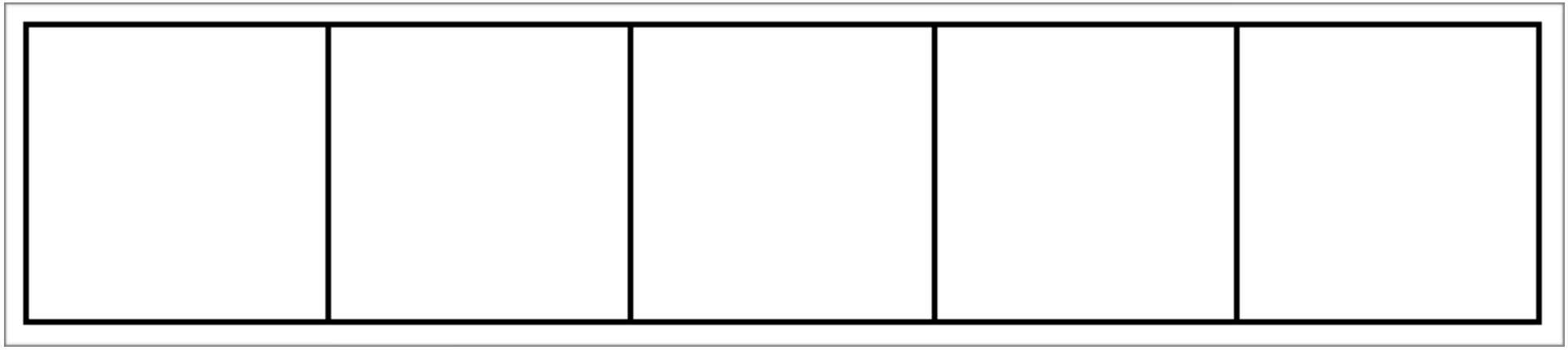


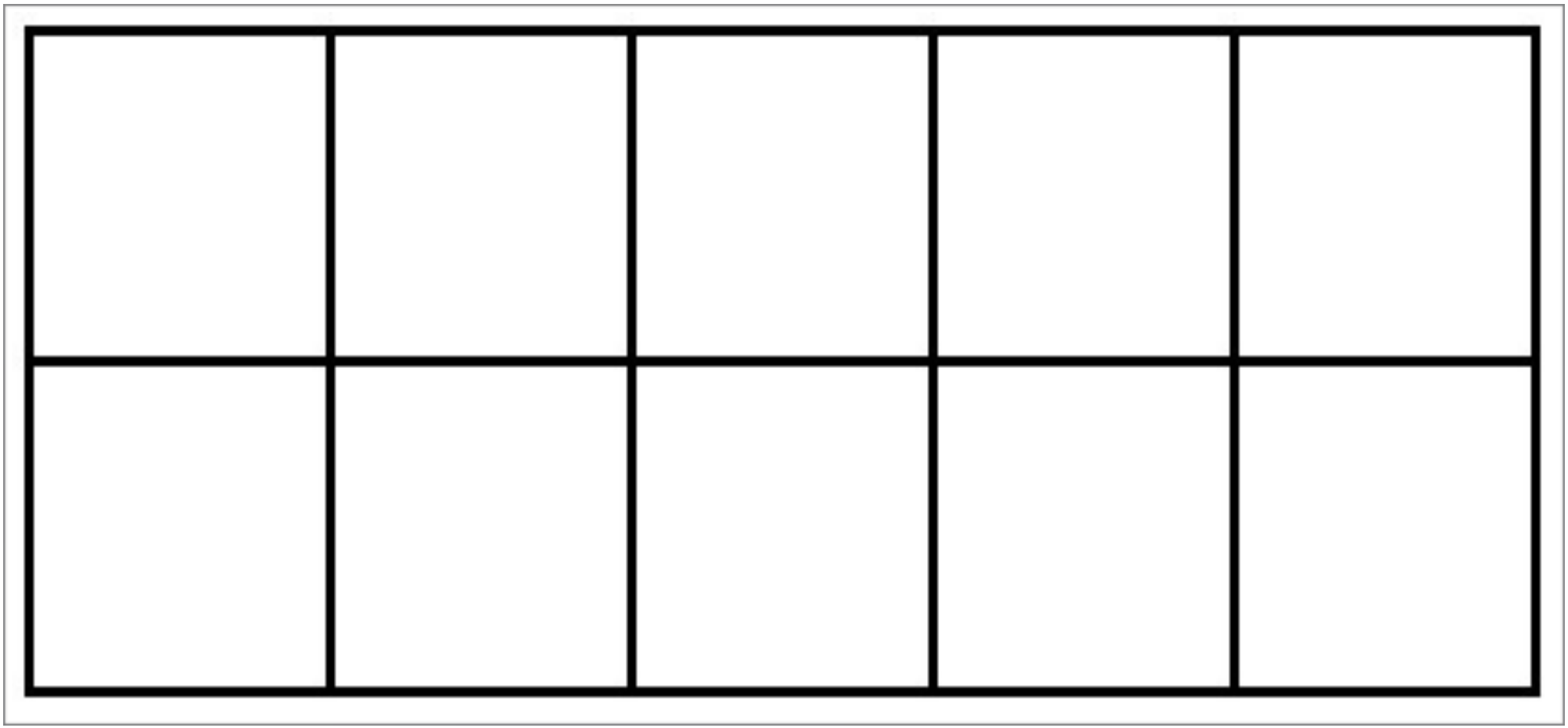
Blackline Master #8 - One/Two More and Less Mat

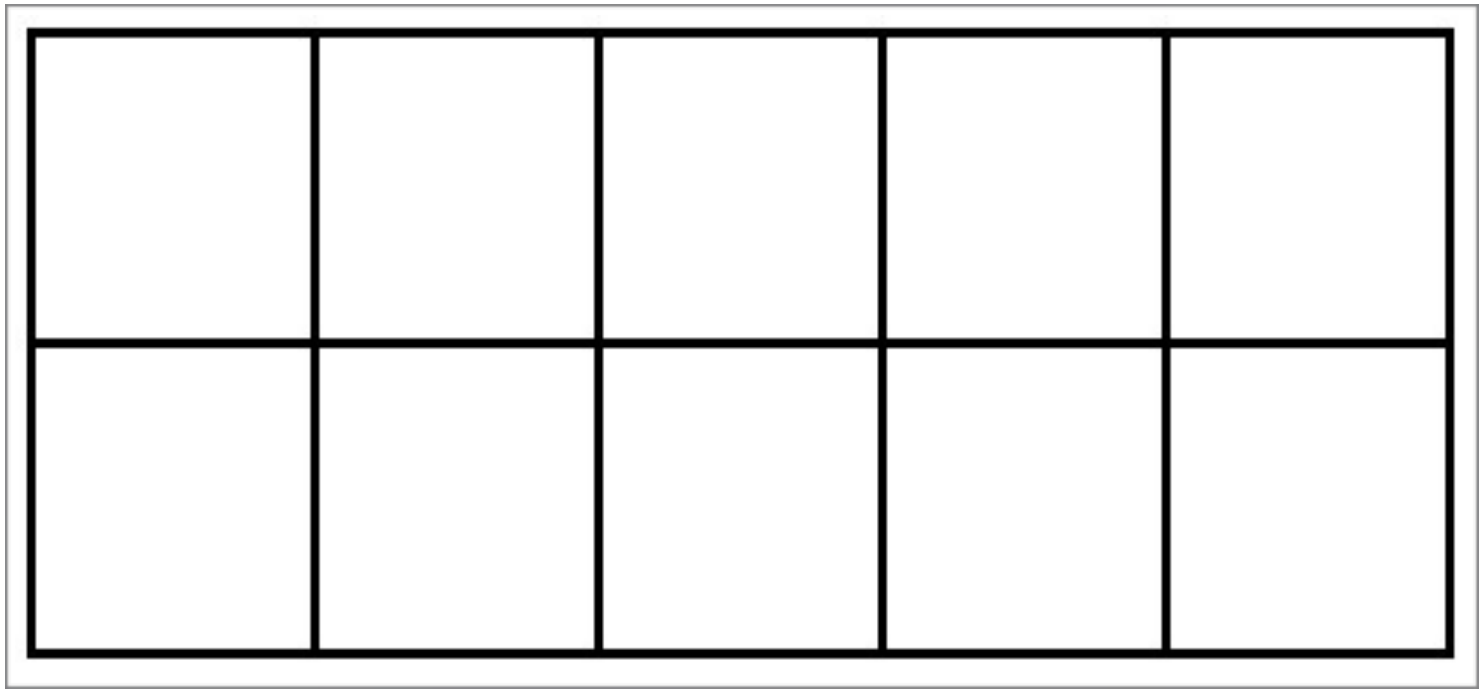
My Scoop	
One Less	**One More**

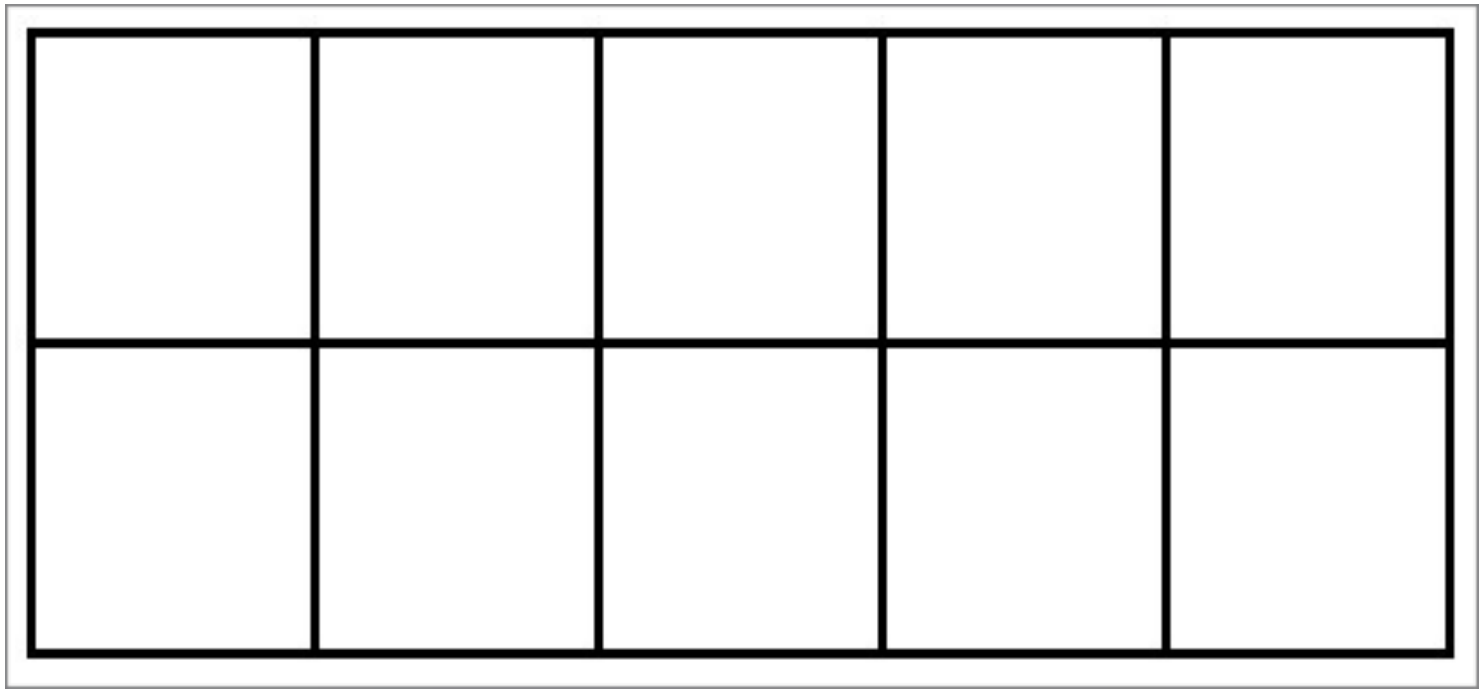
My Scoop	
Two Less	**Two More**

Blackline Master #9 - Five and Ten Frame Mats





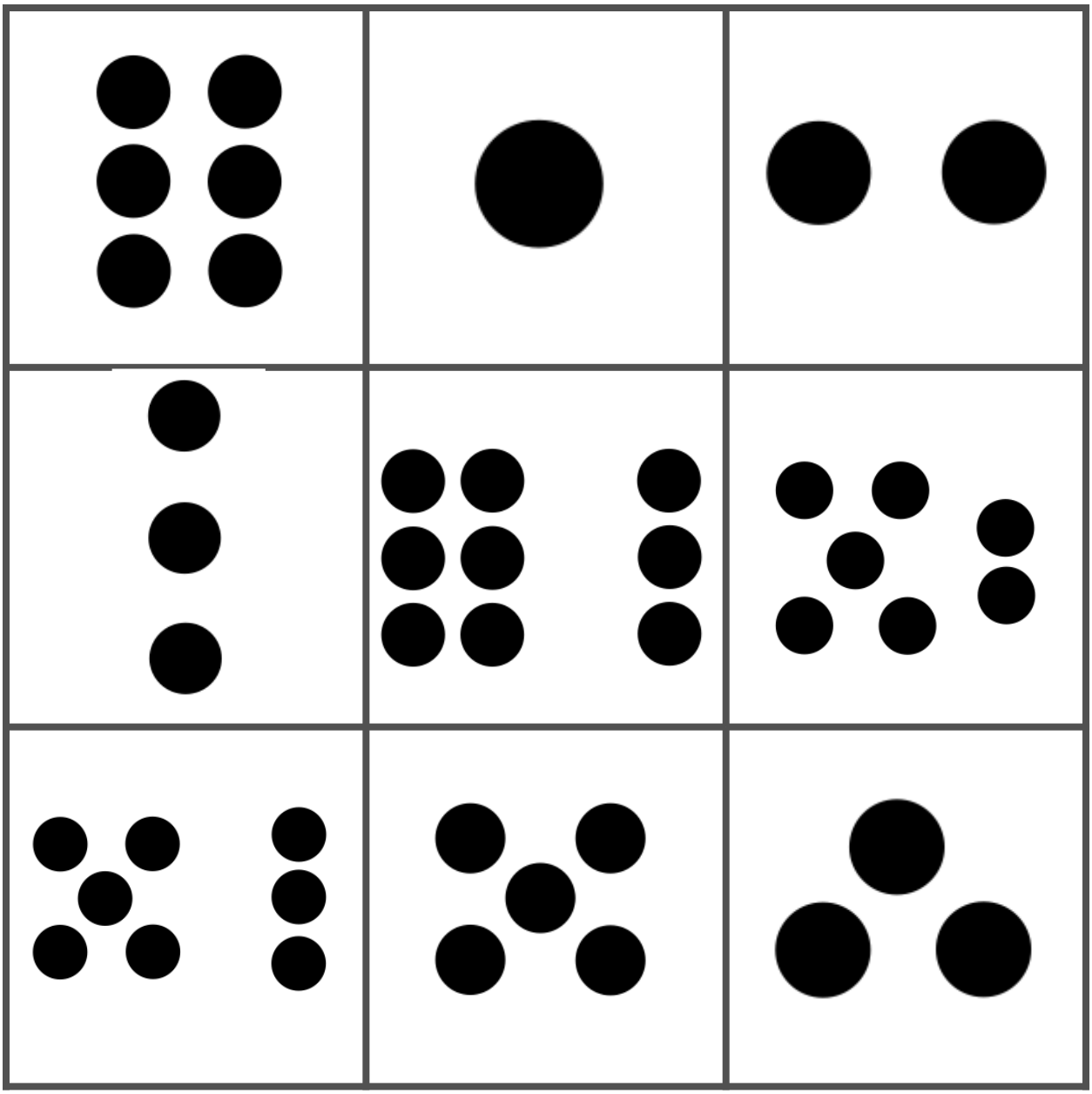


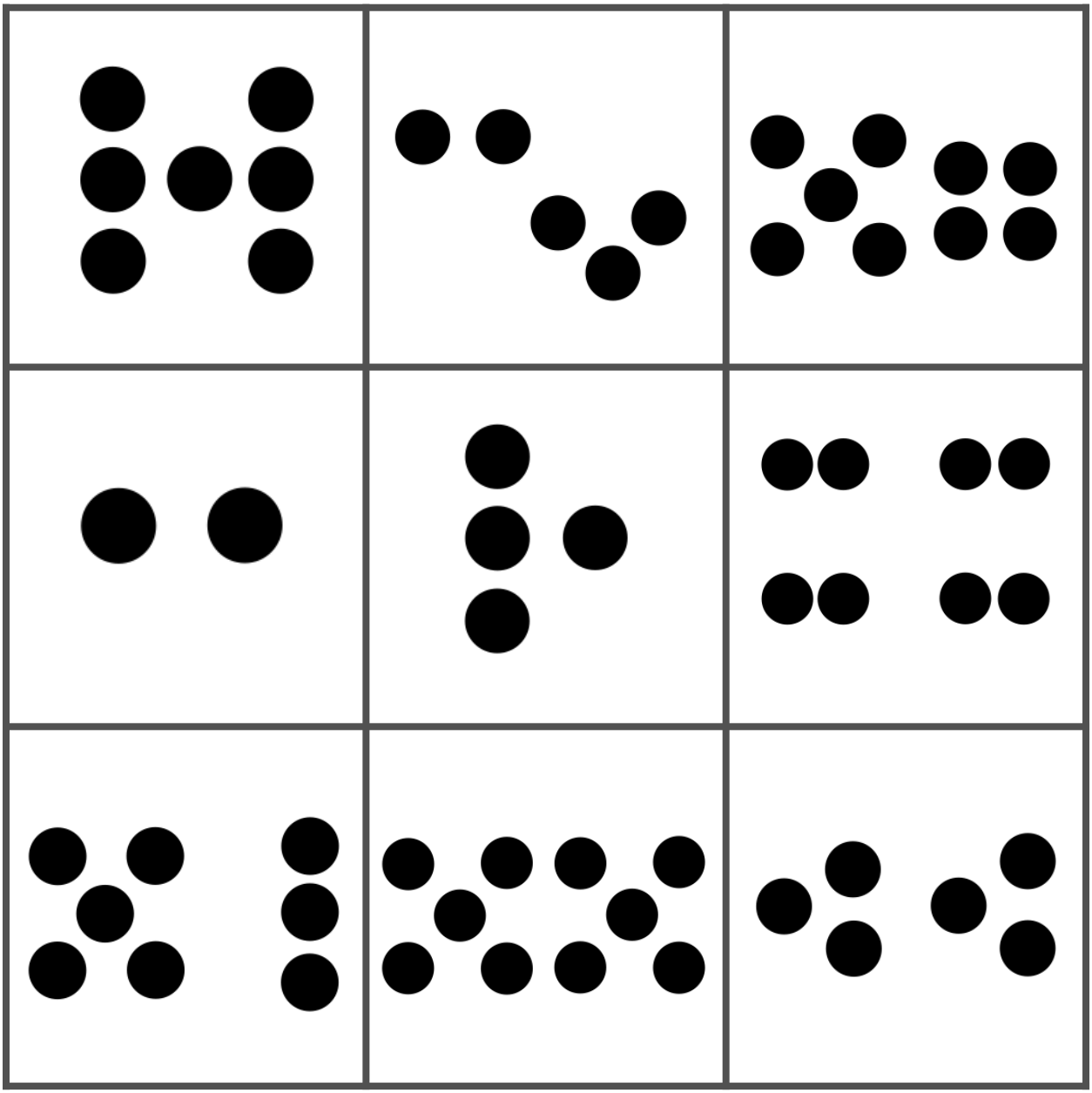


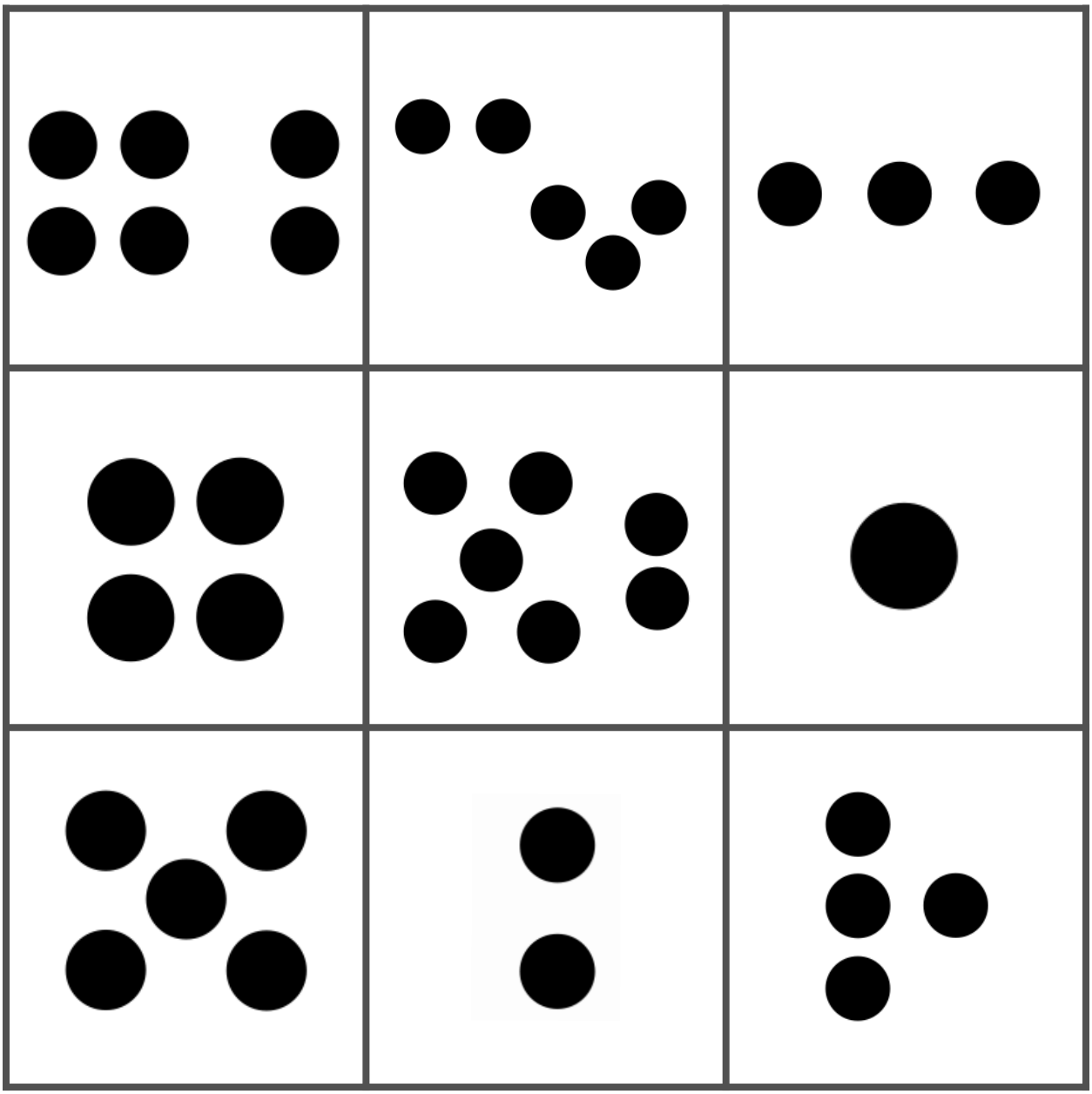
Blackline Master #10 - 'I Choose' BINGO Boards

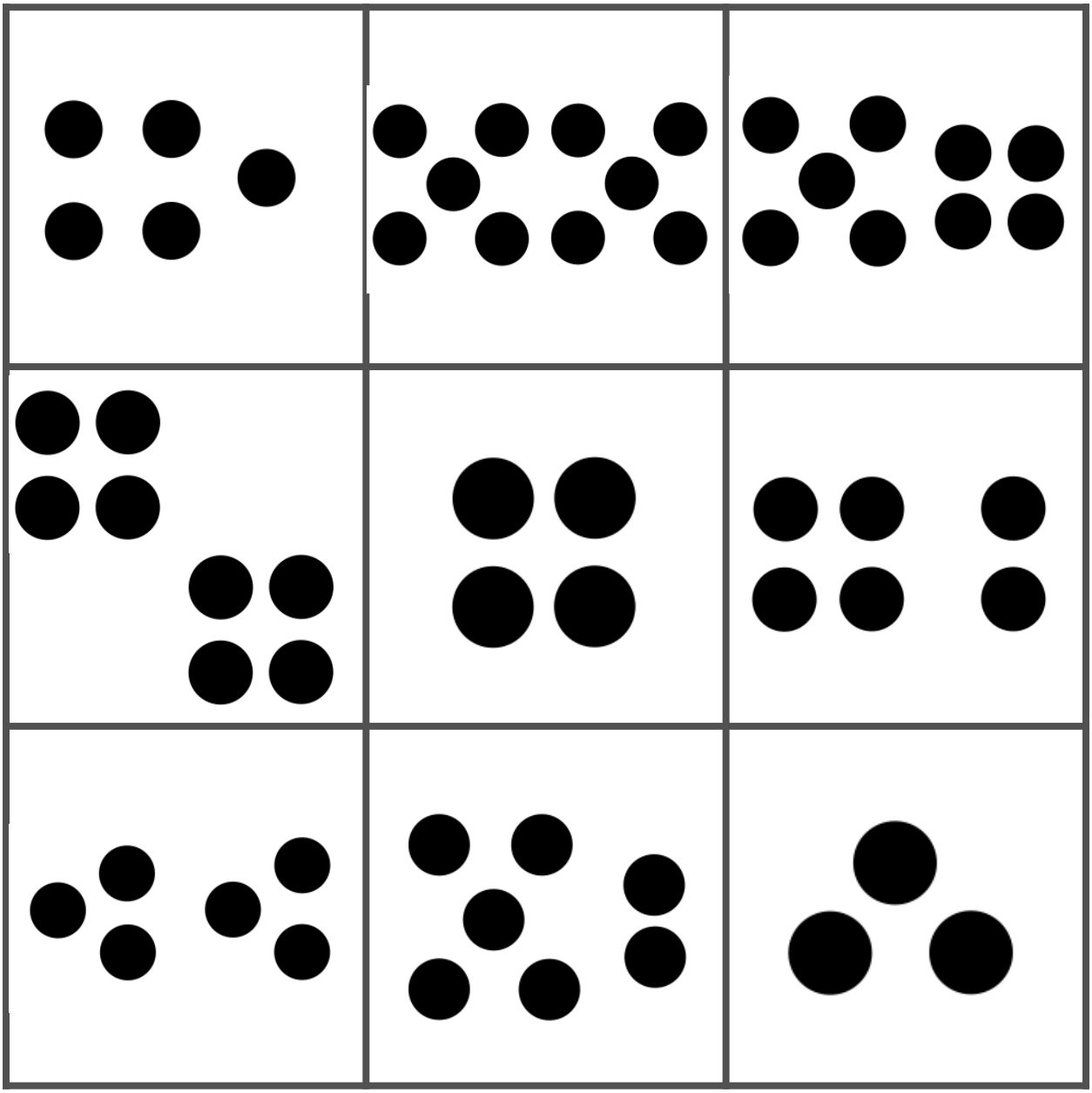
There are various BINGO boards in this document. Use the ones that are appropriate for the children. Some children need the support of the dot patterns before moving into numerals. Some children are ready to move onto larger numbers, while others need small numbers.

There four boards of each type. They are meant to be used in a math center with a small group, but if you want to use it as a whole class you can make multiple copies of each board. Children get to choose if they want to select the number that is more or less than the number that actually gets drawn, so not every child will choose the same numbers.









1-10 Numeral Cards

4	1	3
2	6	5
5	9	7

8	6	2
5	3	6
4	10	9

5	1	4
2	6	7
3	4	8

6	3	8
3	5	10
9	2	4

10-20 Numeral Cards

14	13	17
10	18	15
19	17	16

12	14	16
19	17	12
13	20	15

19	10	12
16	11	17
18	12	13

11	18	19
13	15	20
12	14	13

Blackline Master #11 - Number Paths

Many kindergarten and 1st grade classrooms have a number line along the classroom wall and on every child's desk. However, **research has shown that *number lines* are conceptually too difficult for young children and instead we should be using *number paths* until a child is in 2nd grade (Fuson, et.al., 2009**). A number path is a count model; the numbers are represented by a rectangle and each rectangle can be counted. A number line is a length model; each number is represented by its length from zero. With a number line, children have to count the length units and not the numbers. The number line model makes it difficult for young children to see the units whereas in the number path the units are easy to recognize. With a number line, some children start their count with zero and are off by one. Some count the space between the numbers and when they end their count their finger is between two numbers (count 4 spaces and your finger is between the 3 and 4) so they are unsure which number to use. Both of these errors do not occur when using a number path. Young children are still making sense of numbers, we do not want to use models that confuse them; instead we want models that help solidify and build their understanding.
Jerome Bruner (1966) identified a progression that children go through that helps them make sense of the formal symbolism we use in mathematics:

Enactive – using tangible items to model the problem; the MathRack, ten frames, cubes, acting it out, etc.
Iconic – representing what they did in the enactive phase with an icon (tally marks, circles, etc.) on paper. This could be drawing it or showing it on a number path.
Symbolic – writing the formal signs and symbols

It is important that we allow children to solve problems in the enactive phase, but we do not want them to stay in that phase forever. As they experience the problem tangibly, we want to show them ways to notate that on paper which reflect what they did in the enactive phase. For some children we need to be very explicit about helping them see the connection between what they notated on paper and what they did with the manipulative. I also have them write the numeric version of what they did. This helps them make the connection between all three representations of the problem (enactive, iconic, and symbolic).

Whenever you use manipulatives in your classroom you have to be cautious that the manipulative does not become a crutch. The number path and number line are powerful tools that give students the iconic bridge to make the leap from the manipulatives we use to the writien symbols. Manipulatives are an excellent way to help build students' conceptual understanding. Unfortunately, when we use manipulatives continually without helping students connect them to the formal mathematical ideas we need them to understand, the manipulative can become a crutch that they feel they need in order to solve problems.

The next page of Number Paths are designed to be printed on 11x17" paper.

You can laminate them and have the students use dry erase crayons to show their thinking. The dry erase crayons will wipe off. Or tape the number path to desks with packing tape covering the number path, and whiteboard marker will wipe off the packing tape.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Broken Number Paths

1	2		4	5	6		8	9
2	3	4		6	7		9	10
	4	5	6	7	8	9	10	
4	5	6		8	9	10		12
5	6		8	9	10			13
	7	8	9	10		12		14
	8	9	10		12		14	

12		14		16	17	18	19	
13	14		16	17	18	19		
14		16	17		19		21	22
	16	17	18	19	20		22	23
16	17	18	19			22	23	24
	18	19	20		22	23		25
18	19			22	23	24	25	

8	9			12	13	14	15	
9			12	13	14	15		17
	11	12	13	14	15		17	
11	12		14		16	17	18	

19			22	23	24	25		27
	21	22	23		25	26		28
3		5	6	7	8	9		
6	7	8	9				13	14

14		16	17	18	19			22
	16	17	18	19			22	23
17	18	19			22	23	24	
	20	21	22	23	24			27

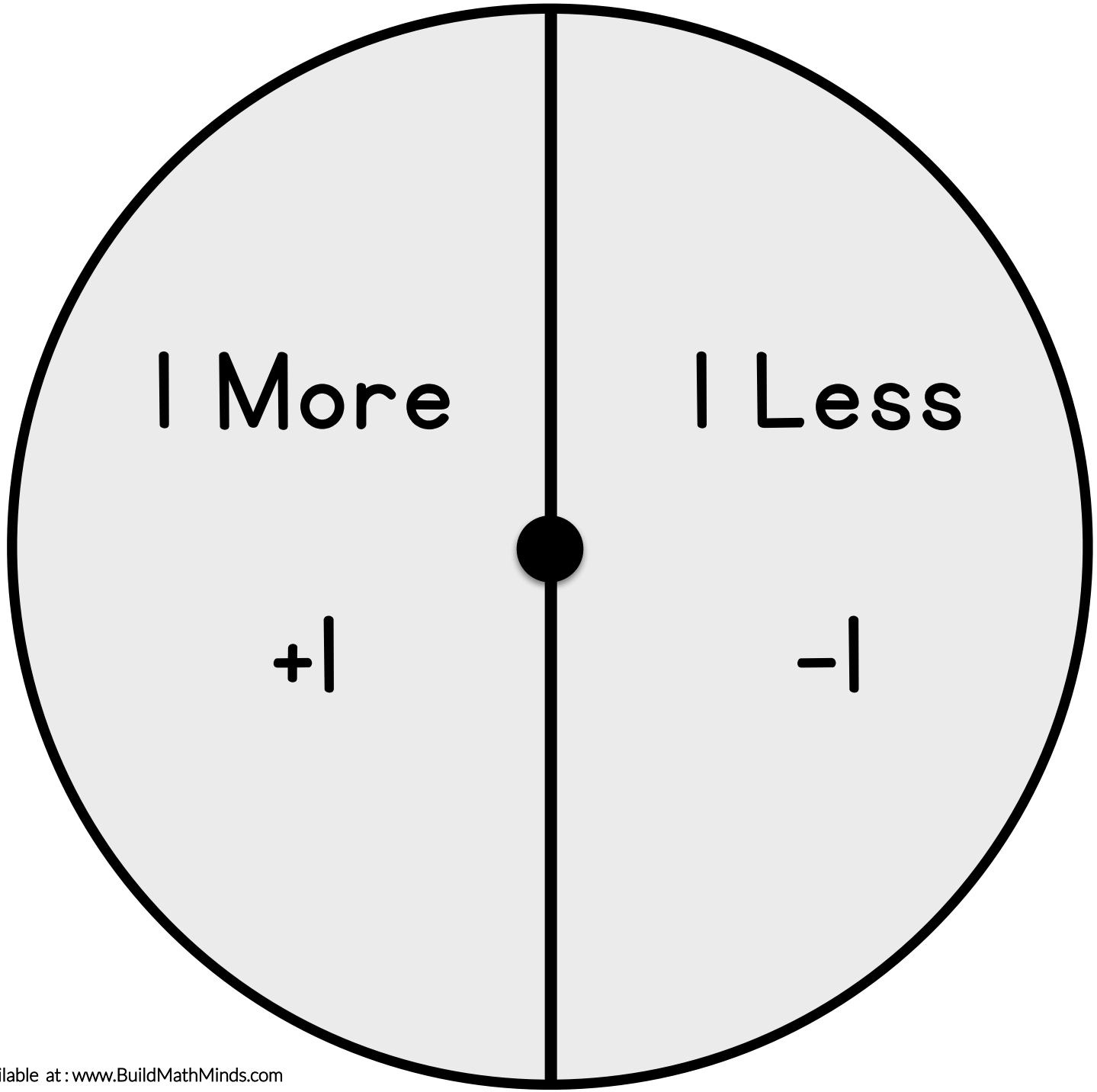
8	9				13	14	15	16
	6	7	8	9	10			13
		13	14	15	16		18	19
12		14		16	17	18	19	

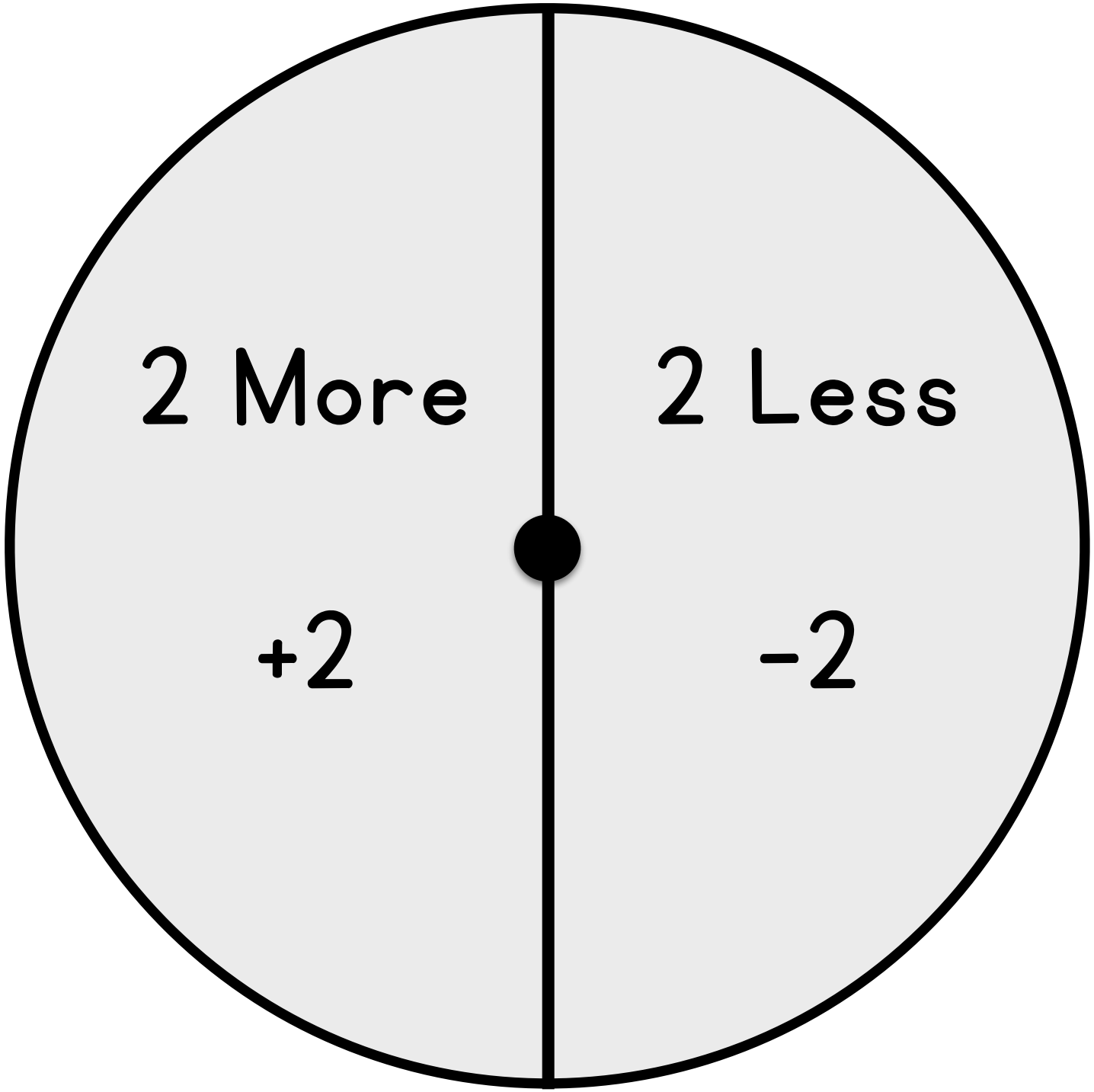
Teen Number Paths

11	12	13	14	15	16	17	18	19

11	12	13	14	15	16	17	18	19

Blackline Master #12 - More/Less Spinner




2 More
+2
2 Less
-2

Blackline Master #13 - Place Value Cards

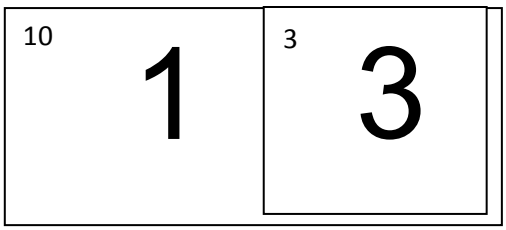
These Place Value Cards allow children to see the value of each digit within a number. The small numbers in the upper left-hand corners of the cards shows the expanded notation to remind children that the "3" is really worth a "30." The back of each card shows the MathRack's visual representation of each number, to help children build spatial relationships for numbers. A set contains one card each of the numbers 1-9, the decade numbers (10, 20, 30…), and a 100 card. This allows children to make all the numbers from 1 to 199. Dimensions when you cut the cards should be: 3x2" for 1-9, 3x4" for 10-90, 3x6" for the 100 card.

This document is designed to print the MathRack picture on the back of each numeral card. The cards can be used without the MathRack products, but I highly recommend the MathRack tools as an additional way to reinforce place value ideas. You can use the Place Value Cards in lots of activities, but the general idea is to have the children build numbers and discuss the "hidden value" of the digits. Below are directions on how to use the Place Value Cards along with the MathRack tools. You can adjust the lesson if you do not have them, or go to www.mathrack.com to learn more about MathRacks.

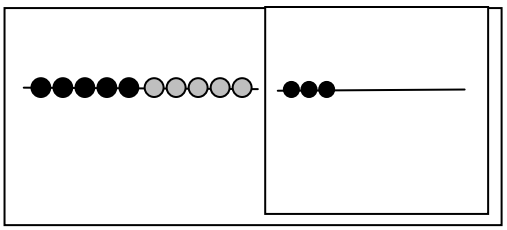
Using the Cards:

- Have a child build 13 using the MathRack 10 Rows of 10 or the Add-A-Row**.** Ask the class to write that number down. Next, ask them to show that number using the place value cards (example below). Once they have their cards put together, have them flip the cards over to see the MathRack representation on the back. Is the picture on the back the same as their MathRack? **Teacher suggestion**: Because you can only see the MathRack picture of the ten when you flip the cards over, pair up the children and have one of them show the number with their place value cards and the other show the picture.

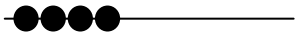
Front Side

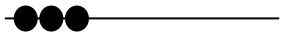


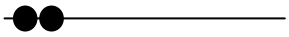
Back Side

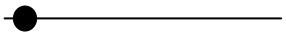


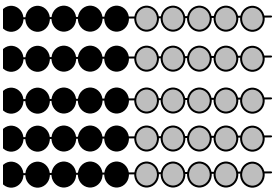
- As a group, come to a consensus about how to show 13 with the place value cards (10 and a 3). Talk about what is hiding when we put the cards like that (the 10 is there but it is hidden behind the 3) and the same thing happens when we write the number 13, it really is a 10 and 3. Write the problem as an equation also; 13 = 10 + 3
- Do more numbers (11-100) with them, always having them show it on the MathRack, writing the numeral, and then showing it with the place value cards to check their understanding of the written numeral.
- Once children are comfortable with this activity, try reversing it. Say a number out load, ask them to make the number using place value cards, and then show it on the MathRack to check that the picture on the back of their cards matches what they build on the MathRack.











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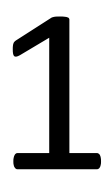


2

3

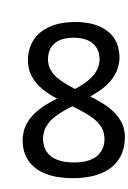
4

5

6

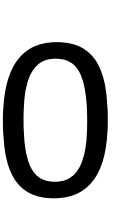
7

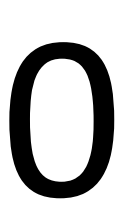
8

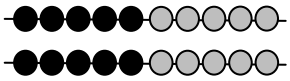


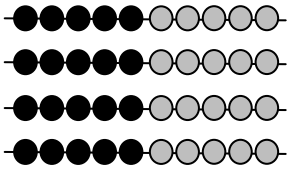
9

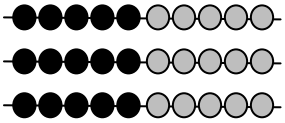
100

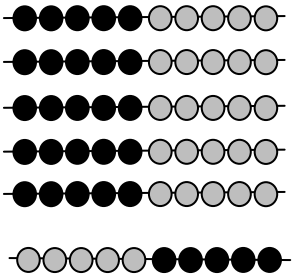


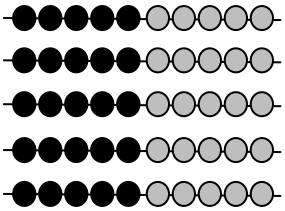












10

1 0

20

2 0

30

3 0

40

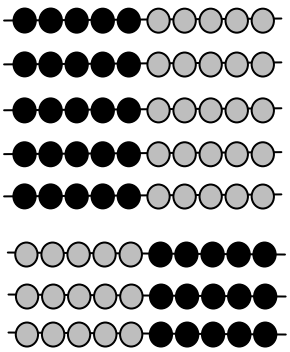
4 0

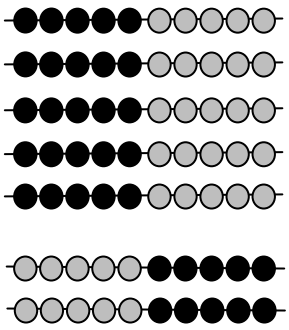
50

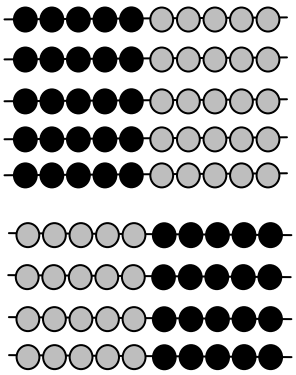
5 0

60

6 0







70

7 0

80

8 0

90

9 0

Blackline Master #14 - Teen Missing Part Cards

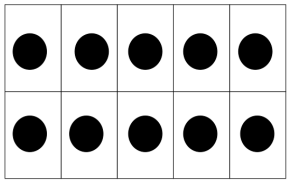
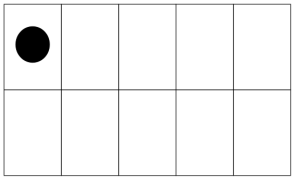
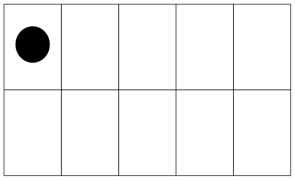
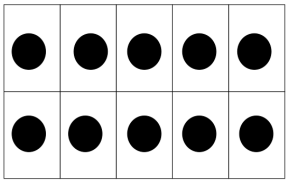
The idea for these cards comes from an activity in John Van de Walle's book *Teaching Student Centered Mathematics PreK-2*. Missing Part Cards help develop Part-Part-Whole relationships. Typical Part-Part-Whole activities give children the two parts and ask them to find the whole, however, Missing Part Cards give the children the whole (the numeral) and one of the parts (in this case as a ten frame and some have the numbers instead of ten frames) and kids are asked to find the "missing part." Missing Part activities are more difficult than traditional part-part-whole activities because they help start the progression towards subtraction. We want children to relate these types of problems to addition when they first start out; *what do I need to add to get me up to that total amount?* The best way to learn subtraction facts is to relate them to addition facts.

How to use the cards:

- Each card has a numeral for the whole and a ten frame representation for each part.* Cover one of the ten frames with a flap of paper (I use the small post-it notes so that I can easily change which part is covered).
- Have the child say the missing part that is covered up and/or the number sentence, i.e. "Twelve is ten and two" for a 12 card showing a full ten frame and hiding a ten frame with two.
- Make sure to have the child explain how they knew the missing part.

*Some cards have the 'parts' represented with the numerals instead of ten frames so that you can transition from the ten frame representation to the symbolic form of the numbers.

Another activity, from Van de Walle's book, you can do with the cards is known as "I Wish I Had." You show the child the missing part card, say the amount that is shown on the ten frame that is not covered up, and say that you wish you had the numeral. For example with the 12 card that has a full ten frame showing and hiding two, you would say *"I have 10; I wish I had 12…how many more do I need?"*

11	 
11	 
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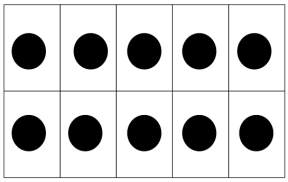
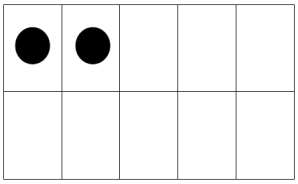
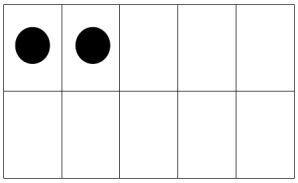
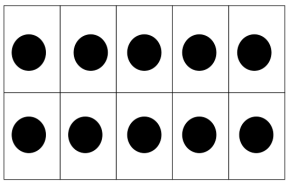
11	1 10

12	 
12	 
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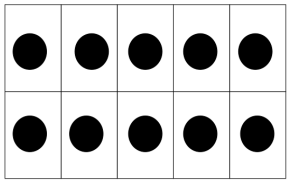
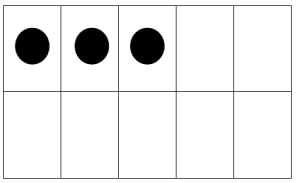
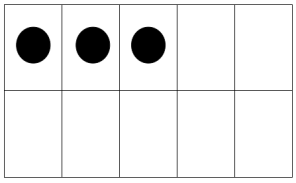
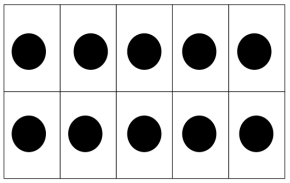
12	2 10

13	 
13	 
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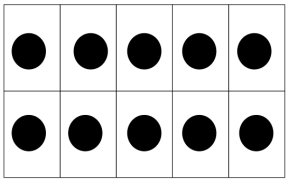
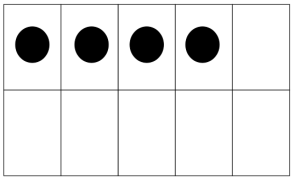
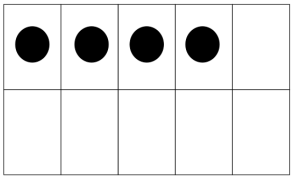
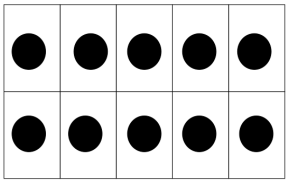
13	3 10

14	 
14	 
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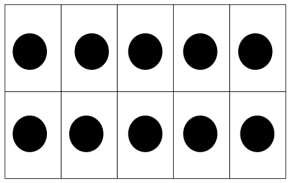
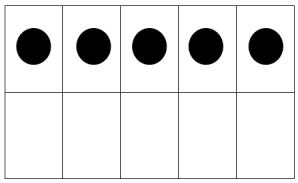
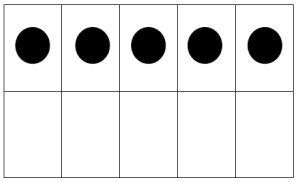
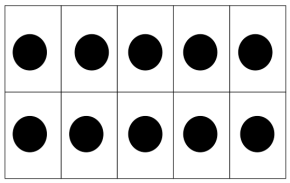
14	4 10

15	 
15	 
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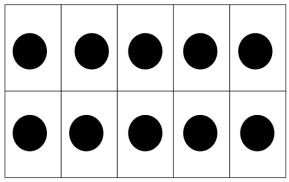
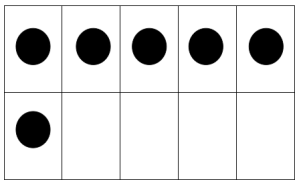
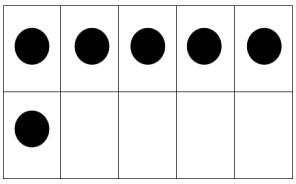
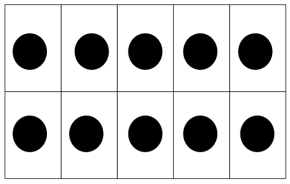
16	 
16	 
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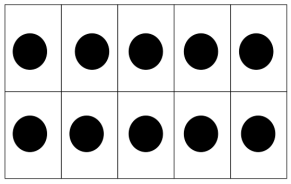
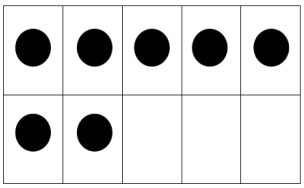
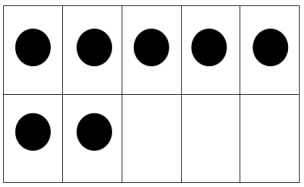
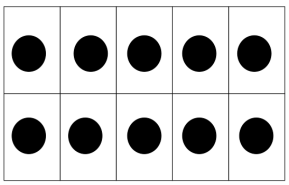
17	 
17	 
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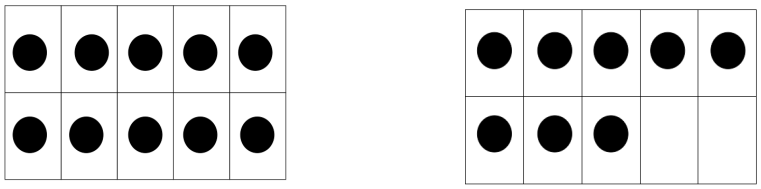
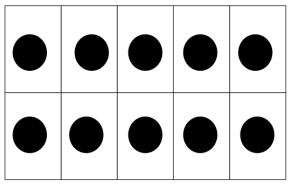
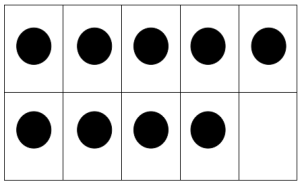
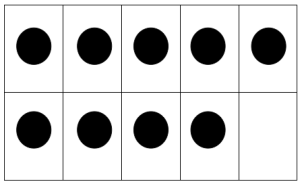
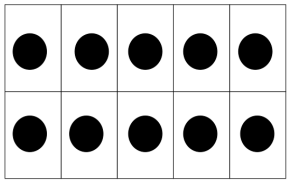
18	
18	
18	10 8
18	8 10

๑๙	 
๑๙	 
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Blackline Master #15 - Shake & Spill Mat

by Christina Tondevold

_____ = _____ + _____ _____ = _____ + _____

_____ = _____ + _____ _____ = _____ + _____

_____ = _____ + _____ _____ = _____ + _____

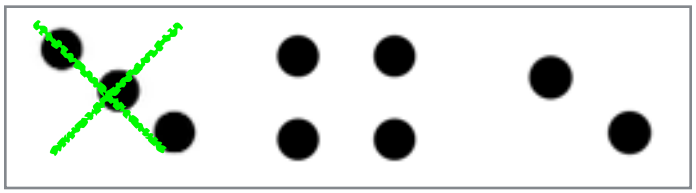
_____ = _____ + _____ _____ = _____ + _____

_____ = _____ + _____ _____ = _____ + _____

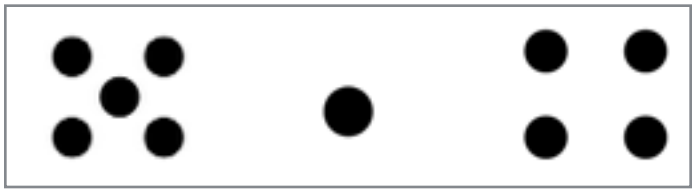
Blackline Master #16 - Two out of Three

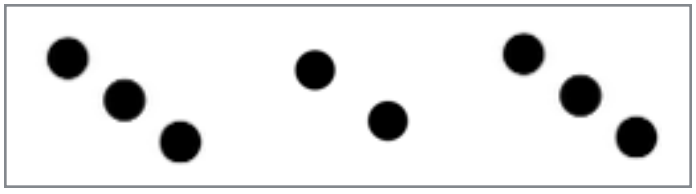
On each page, the number at the top is the "whole" we want the child to make. In each set of three quantities, the goal is to find the two quantities that make the whole. I have them cross out the one in each group that they do not use to make the whole. Then next to each strip, they write the number sentence to describe the part/whole relationship. For example, if they used a 3 and 4 to make 7, they would write 7 = 3 + 4.

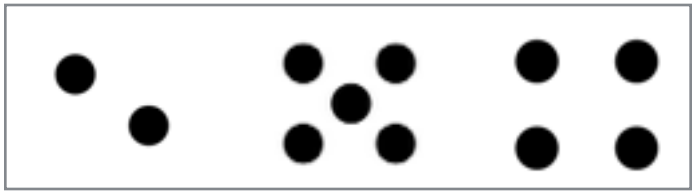
6



6 = 4 + 2







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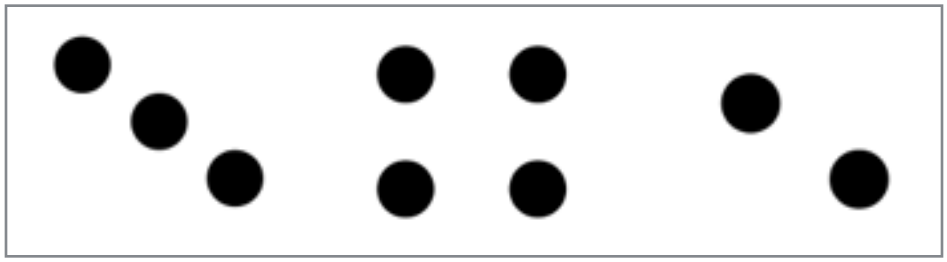

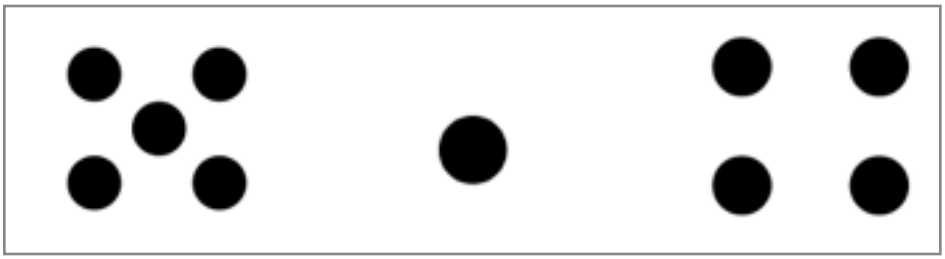

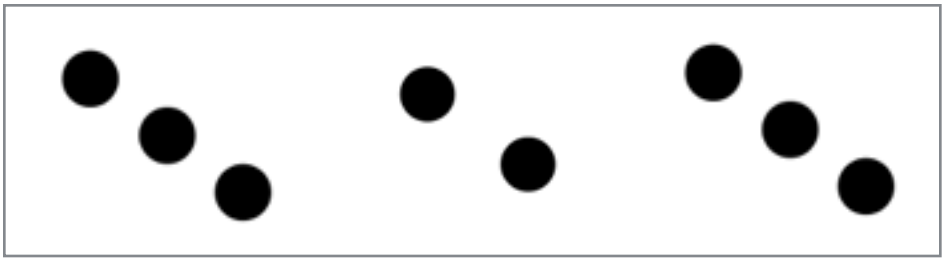

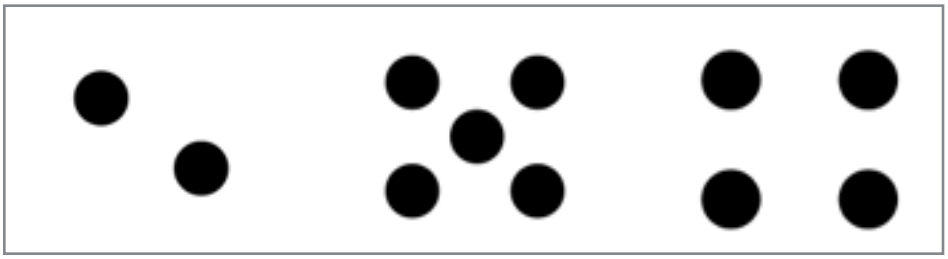

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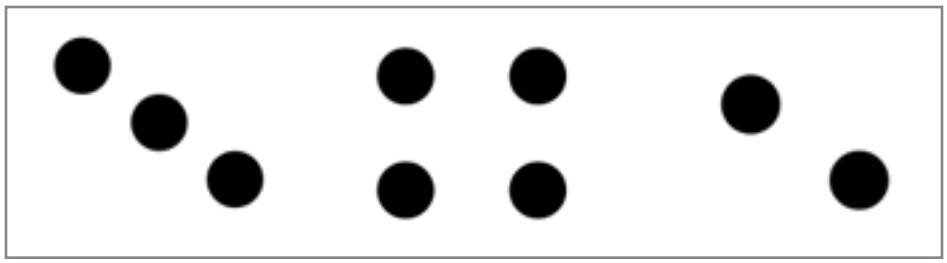

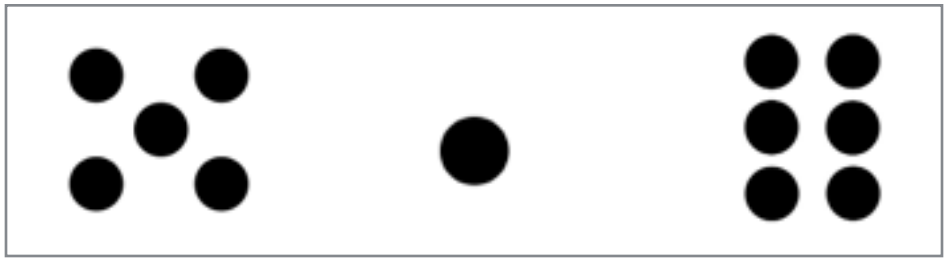

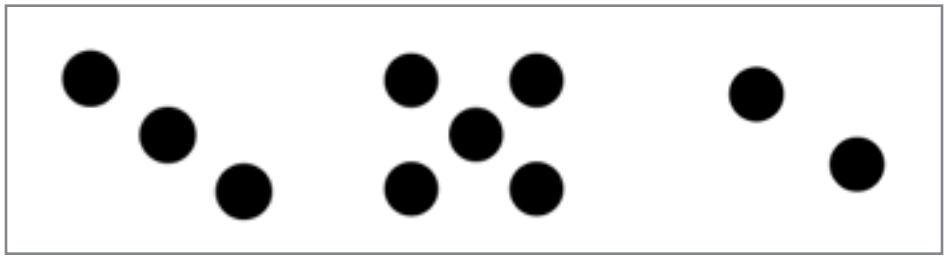

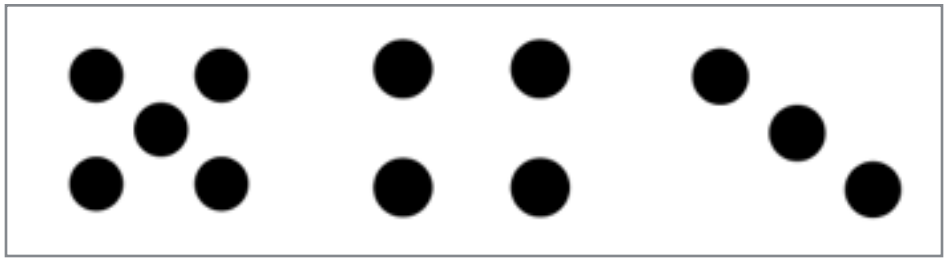

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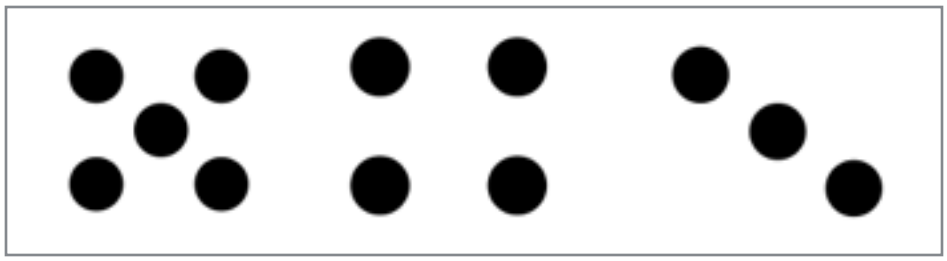

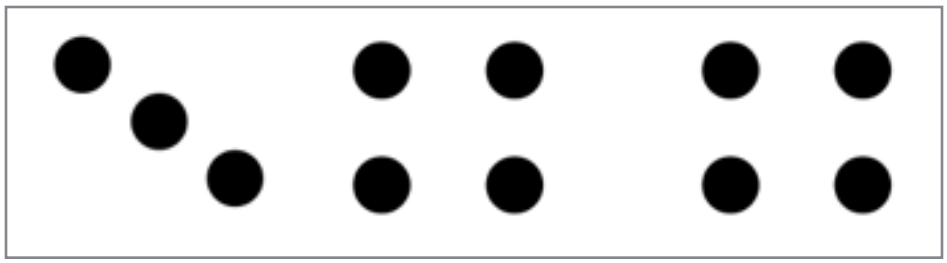

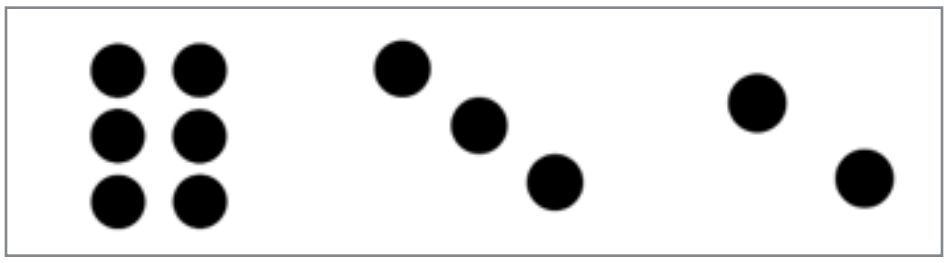

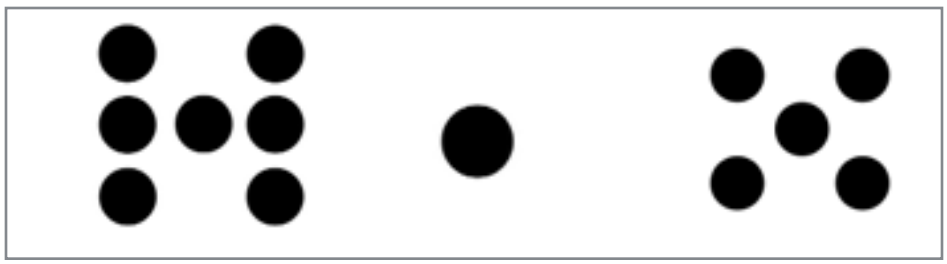

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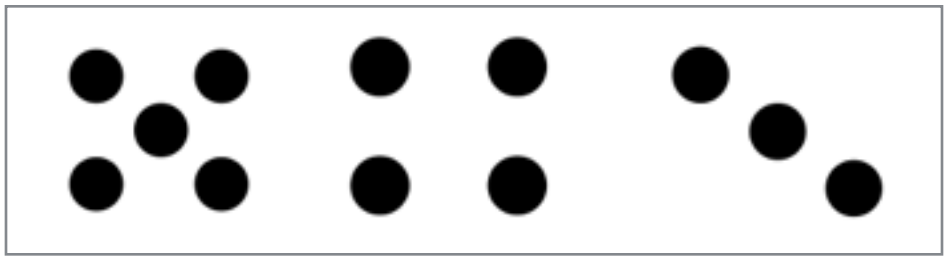

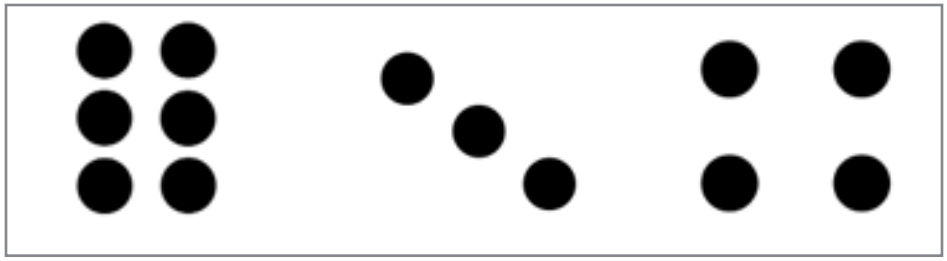

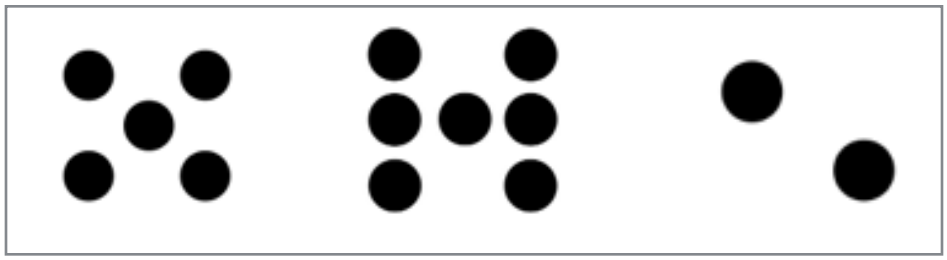

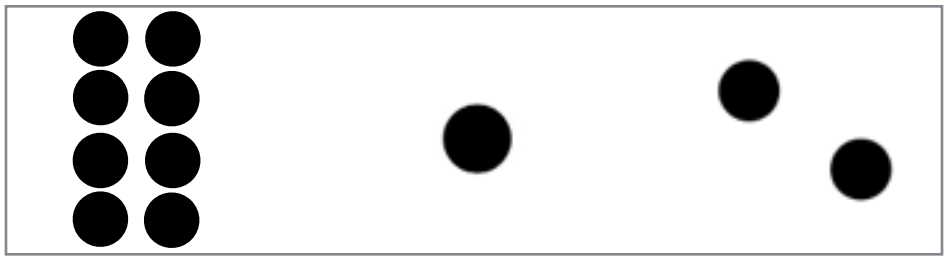

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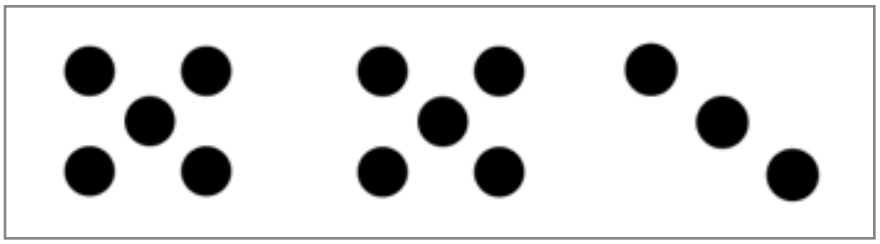

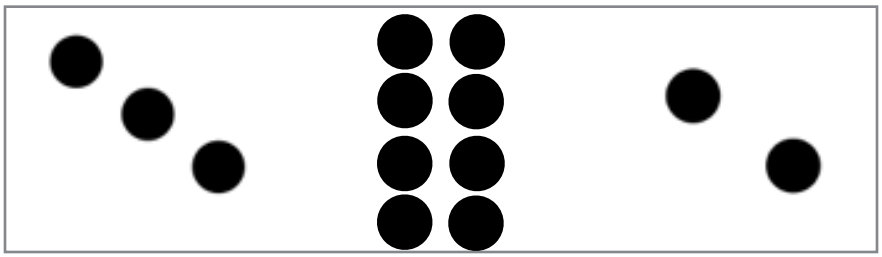

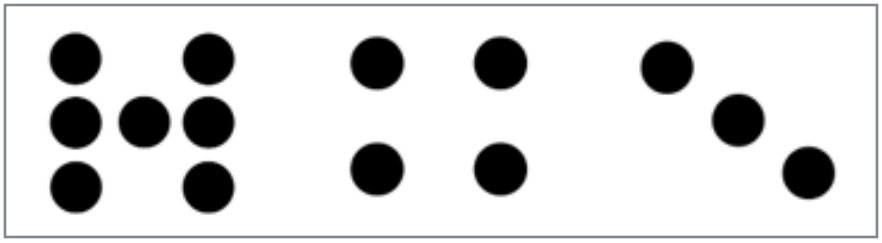

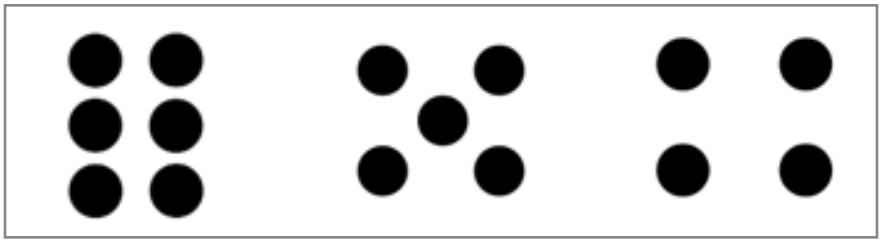

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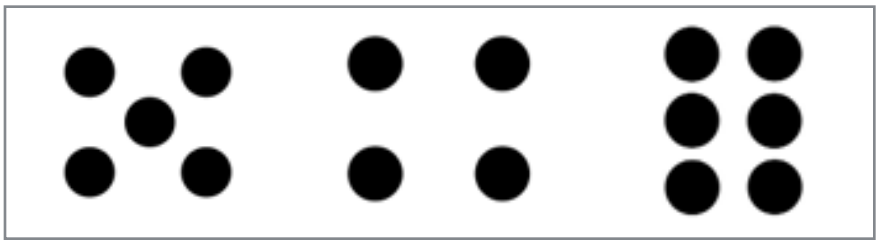
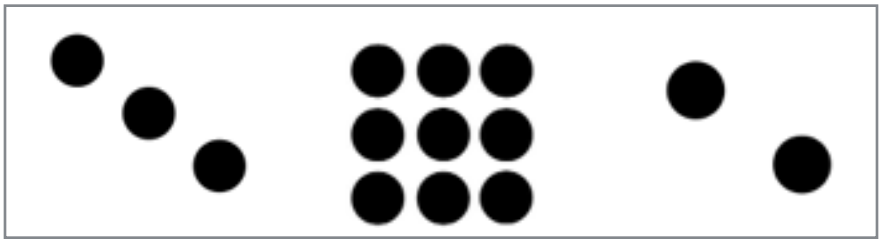
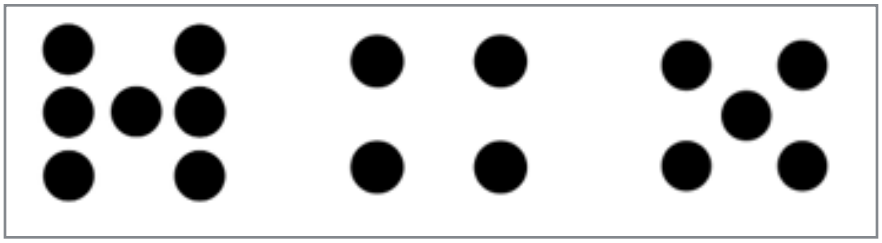
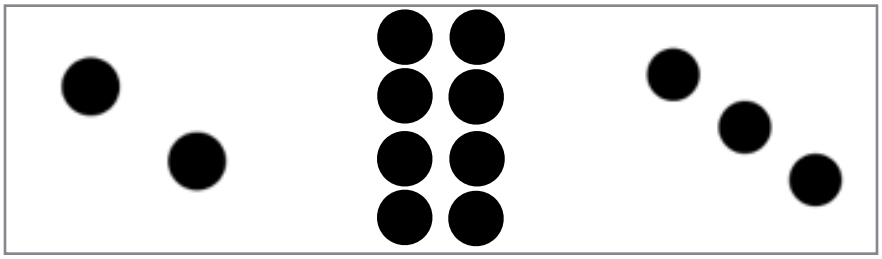








12

6	6	5
4	7	8
3	10	2
7	5	6

13

6	7	8
9	5	4
10	7	3
7	8	5

14

6	7	7
8	5	9
4	10	8
7	8	6

15

10	7	5

7	8	6

6	9	4

8	4	11

16

6	9	10
15	8	1
9	7	6
8	9	8

17

11	5	12
6	8	9
2	3	15
10	7	12

18

9	8	9
7	10	11
13	8	10
15	3	14

19

10	8	9
7	8	11
15	4	5
6	7	12

Blackline Master #17 - Missing Part Cards

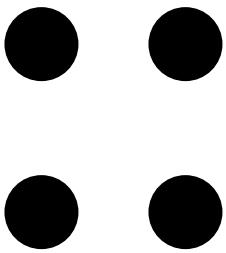
The idea for these cards comes from an activity in John Van de Walle's book *Teaching Student Centered Mathematics K-3.* Missing Part Cards help develop Part-Part-Whole relationships. Typical Part-Part-Whole activities give children the two parts and ask them to find the whole, however, Missing Part Cards give the children the whole (the numeral) and one of the parts (as a dot pattern) and kids are asked to find the "missing part." Missing Part activities are more difficult than traditional part-part-whole activities because they help start the progression towards subtraction. We want children to relate these types of problems to addition when they first start out; *what do I need to add to get me up to that total amount?* The best way to learn subtraction facts is to relate them to addition facts.

How to use the cards:

- Each card has a numeral for the whole and a dot pattern for each part. Cover one of the dot patterns with a flap of paper (I use the small post-it notes so that I can easily change which part is covered).
- Have the child say the missing part that is covered up and/or the number sentence, i.e. "Eight is five and three" for an 8 card showing five dots and hiding three.
- Make sure to have the child explain how they knew the missing part.

Another activity, from Van de Walle's book, you can do with the cards is known as "I Wish I Had." You show the child the missing part card, say the amount of the dot pattern that is not covered up, and say that you wish you had the numeral. For example with the 8 card that has five dots showing and hiding three, you would say *"I have 5; I wish I had 8…how many more do I need?"*

1	
2	
2	
3	

3	
4	
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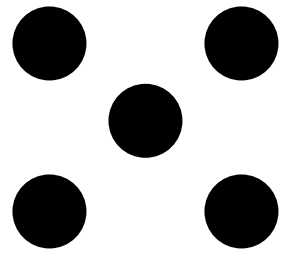
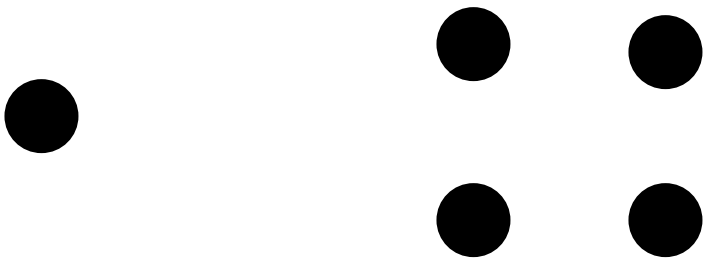
5	
5	
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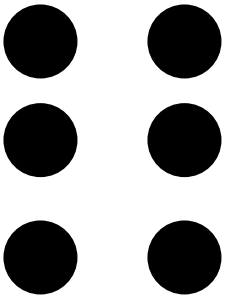
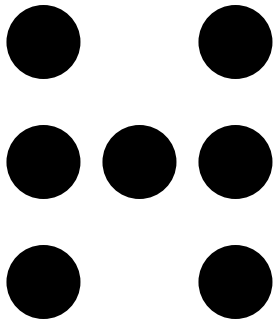
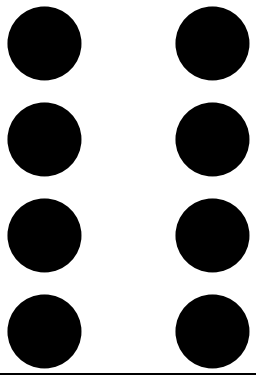
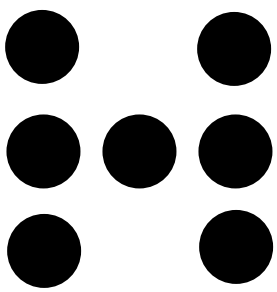
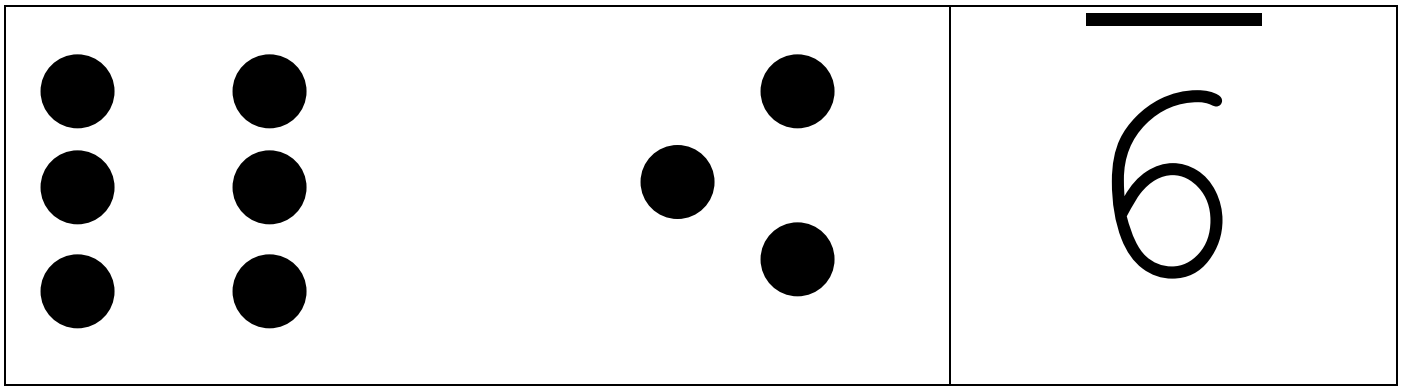
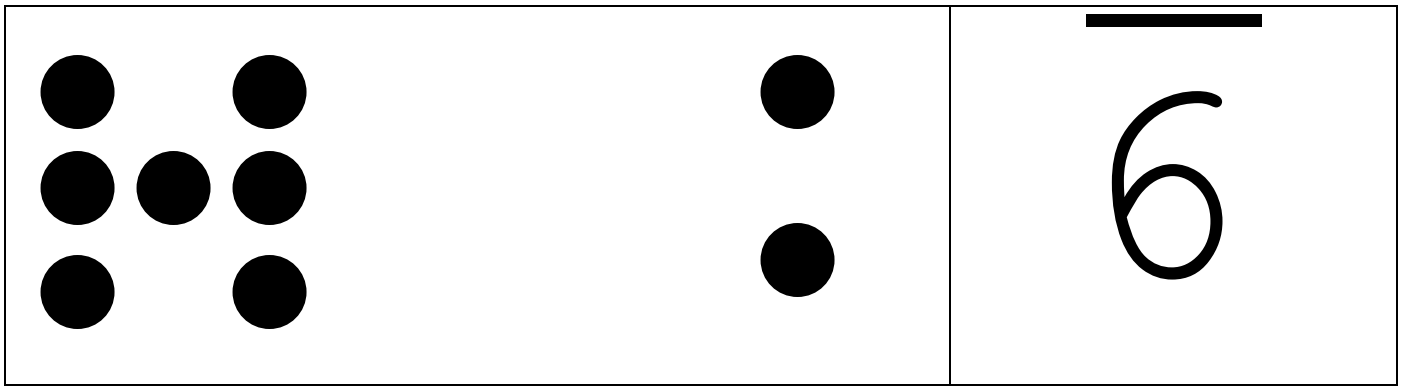
6	

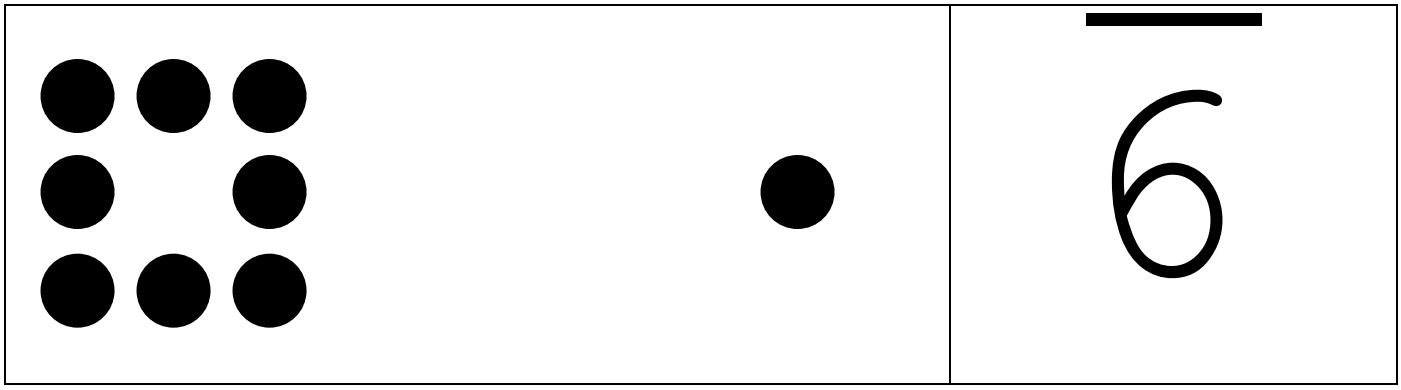
6	
6	
6	
7	

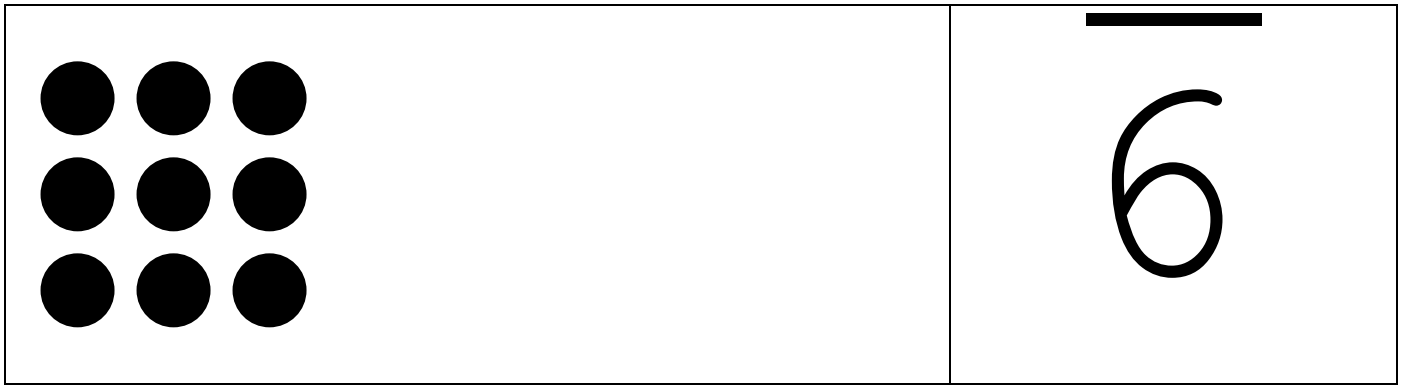
7	
7	
7	
8	

8	
8	
8	
8	









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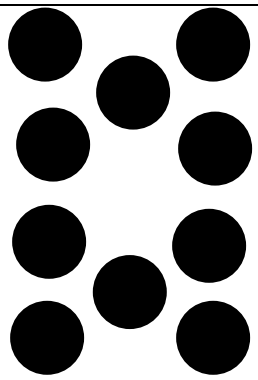
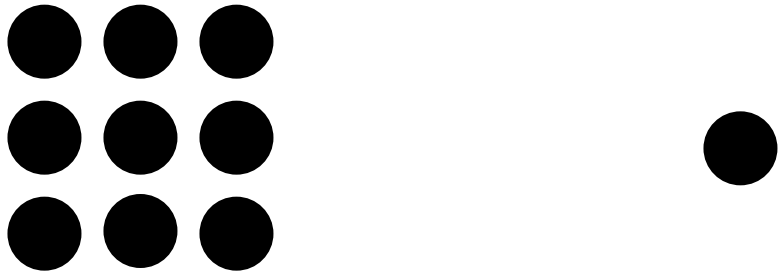
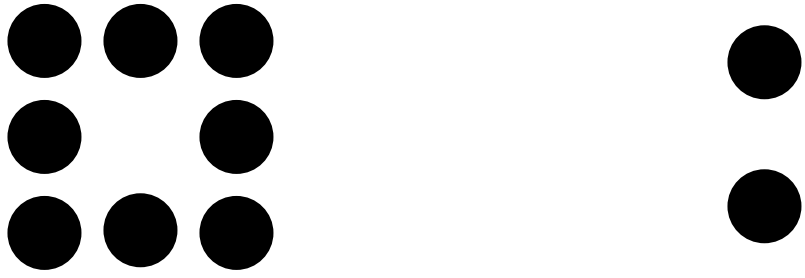
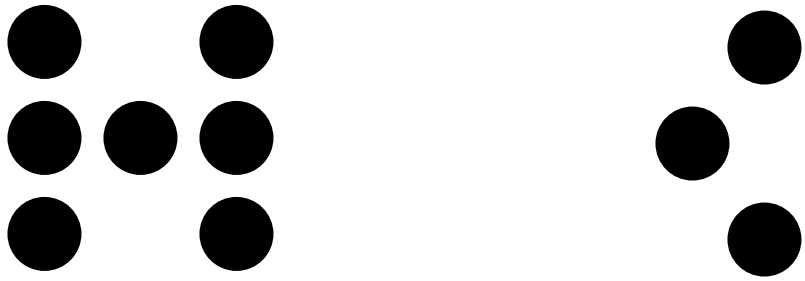
10	
10	
10	

10	
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Blackline Master #18 - Number Search Sheets

There are three different Number Search Sheets. The first one is for when the 'whole' is the numbers 1 through 5. The second sheet is for when you want the whole to be one of the numbers 6 through 10. The third sheet is created to have the numbers 11 through 20 as the whole.

Sums 1 through 5

3	1	4	3	2	1	1	0
0	3	1	2	3	2	4	5
2	2	1	4	2	0	1	3
1	0	3			1	3	2
4	1	2			4	1	3
3	5	2	1	3	0	2	0
1	0	3	4	1	3	2	1
4	2	1	3	2	4	0	5

Sums 6 through 10

3	6	4	3	0	9	1	0
8	2	4	5	2	7	2	5
2	7	1	9	4	6	4	3
10	0	6			6	3	2
3	4	2			8	1	7
3	5	9	1	6	0	5	2
6	0	7	2	8	3	6	4
2	3	4	2	1	5	5	3

Sums 11 through 20

3	9	4	8	7	6	8	9
7	4	7	8	5	6	4	5
7	2	9	5	9	9	7	3
8	9	3			10	3	6
4	10	2			4	8	7
9	5	7	6	8	6	5	10
7	6	8	4	5	7	6	10
8	9	3	9	6	8	5	7

Blackline Master #19 - Part-Whole BINGO

There are four boards for each set of sums. They are meant to be used in a math center with a small group, or playing with a partner, but if you want to use them for a whole class, you can make multiple copies of each board. Children get to choose which spot they want to select, so not every child will choose the same spot for the same sum.

Sums to 5

4 + 1	2 + 2	2 + 1
4 + 0	2 + 0	3 + 2
0 + 3	1 + 1	3 + 1

1 + 1	2 + 3	3 + 1
3 + 0	2 + 1	2 + 2
0 + 4	1 + 2	0 + 1

2 + 1	1 + 2	0 + 1
4 + 1	3 + 0	3 + 1
1 + 1	0 + 2	4 + 1

1 + 3	0 + 2	2 + 3
1 + 0	2 + 2	3 + 0
1 + 4	1 + 2	1 + 1

Sums to 10

5 + 1	5 + 5	2 + 6
4 + 5	5 + 3	2 + 5
7 + 3	3 + 3	3 + 6

4 + 4	5 + 4	5 + 1
4 + 6	2 + 5	3 + 5
7 + 2	4 + 2	6 + 1

3 + 4	4 + 2	8 + 1
5 + 1	8 + 0	6 + 4
5 + 2	1 + 7	9 + 1

3 + 3	3 + 4	6 + 3
4 + 3	5 + 4	4 + 4
5 + 3	5 + 5	3 + 7

Sums to 20

4 + 7	10 + 2	7 + 8
8 + 6	6 + 7	9 + 7
9 + 9	10 + 10	7 + 7

8 + 9	6 + 8	7 + 5
4 + 10	9 + 3	10 + 7
10 + 3	9 + 6	3 + 8

8 + 10	7 + 7	9 + 8
9 + 6	7 + 8	3 + 9
6 + 10	6 + 5	4 + 9

10 + 2	6 + 8	9 + 9
4 + 9	10 + 9	3 + 8
9 + 8	10 + 1	8 + 8