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Holistic Assessment in Math Class-Triangulating Data

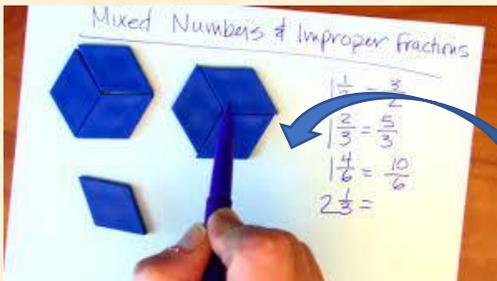
Where do the marks come from that we put on progress reports? Our guiding assessment documents are the CTTCS rubrics. So how do we come up with a number on our 4-point scale (Gr K – 9) or a percent (high school)?

Holistic assessment means we “triangulate data”. We’ve seen this visual many times—but what does it actually mean?

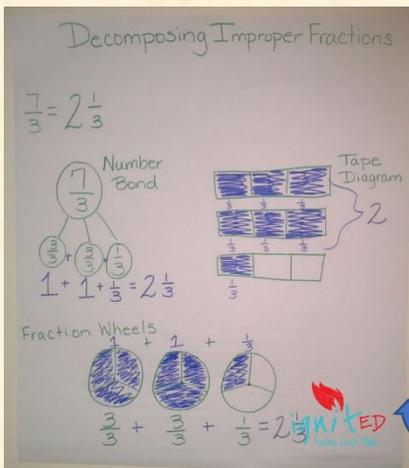
We tend to be “product heavy” in math assessment. It’s easy to collect samples of student work, quizzes, tests, and so on. It’s easy to structure collecting that data. Kids may just sit down and write an assessment. But many of our outcomes require the students to demonstrate **concretely** (with objects, manipulatives) and **pictorially** (by making or analyzing visual representations, diagrams, models) as well as symbolically (by doing the math with numbers, equations, and algorithms). It’s not easy to assess “concrete” and “pictorial” math. Yet to even score at grade level on our rubrics, and the Provincial math rubric, it is necessary.



There is sometimes a misconception that manipulatives and models are scaffolding techniques that help us teach the student the underlying mathematical relationships so that they can then do the math symbolically. Sometimes this is true. For instance, we may show fraction pieces of mixed vs improper fractions, and then have students draw or identify representations.



Concrete (being *connected* to the symbolic representation here)



Pictorial representations or models. Note the variety of models, also connected to number operation and symbolic notation

$$\frac{7}{5} = \frac{5+2}{5}$$
Write the numerator as the sum of two numbers, one of which is divisible by the denominator.

$$= \frac{5}{5} + \frac{2}{5}$$
Write as two fractions. The fraction on the left becomes a whole number.

$$= 1\frac{2}{5}$$
Write the mixed number.

Here's how to convert from an improper fraction to a mixed number.

And finally, **symbolic** representation, including steps of a procedure.

Which of these three images shows the greatest depth of understanding? And which one is the one most often assessed?

One *could* just teach the procedure. We can teach students the steps and have them repeat them. More practice creates more proficiency. But we may have students following the procedure, getting correct answers, and having no concept of what is *actually happening*. Teaching by memorizing procedure creates knowledge that is a house of cards. If a student forgets what they once had memorized, the house collapses. Not only that, there is no room for reasoning, and then applying understanding to future learning and situations. There is no foundation to build on.

“They have been set on the wrong path, often from an early age, of trying to memorize methods instead of interacting with numbers flexibly... Number sense is the foundation for all higher-level mathematics.” -Jo Boaler, Stanford University

Some students will be proficient at working with symbolic representations and algorithms. How often do we train students to “borrow” or “regroup” when they subtract—yet they *have no real concept of what that actually means*. When we ask students to work with base ten blocks, and to remove a certain number, and they have to “trade in” or “make change”, substituting a ten rod for ten unit-cubes, or a hundreds-flat for ten ten-rods, then we connect that concrete action to the algorithm (symbolic form) of subtraction, then a student actually understands “borrowing/regrouping. **To prove their understanding, students must show us more than just the algorithm.**

Here are the grade 8 number outcomes, for example. If we *only* use the SCMA post test, have we really assessed these?

Number Outcomes:

N8.1 Demonstrate understanding of the square and principle square root of whole numbers **concretely or pictorially and symbolically.** ((CN, ME, R, T, V))

N8.2 Expand and demonstrate understanding of percents greater than or equal to 0% (including fractional and decimal percents) **concretely, pictorially, and symbolically.** ((CN, PS, R, V))

N8.3 Demonstrate understanding of rates, ratios, and proportional reasoning **concretely, pictorially, and symbolically.** ((C, CN, PS, R, V))

N8.4 Demonstrate understanding of multiplying and dividing positive fractions and mixed numbers, **concretely, pictorially, and symbolically.** ((C, CN, ME, PS))

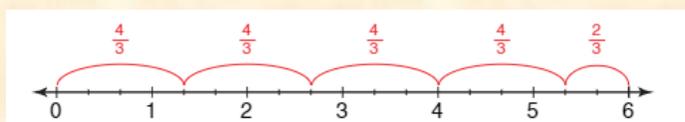
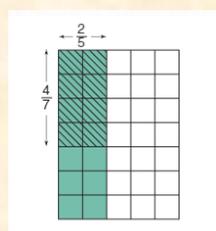
N8.5 Demonstrate understanding of multiplication and division of integers **concretely, pictorially, and symbolically.** ((C, CN, PS, R, V))

Assessed these?

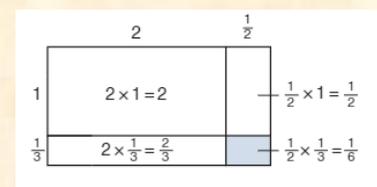
The SCMA is only ONE piece of data contributing to our assessment. **Using SCMA is not a division requirement**, though it may be a school requirement. Check with your administrator.

There is another misconception: that if students can do the work symbolically (solve the equation, multiply the fractions, etc) that they understand the math, so we are done with manipulatives and models. But the truth is, in many cases, the algorithm is the easy part! It's much harder to show your understanding with a model, and if you can, that is *deeper understanding*. Not only that, but many students also get very proficient at memorizing and replicating procedures, and do not necessarily understand the concept. If we only assess the symbolic math, our perception of a student's understanding can be inflated. The notion that manipulatives are the *means* and the algorithmic operation is the *end* is false.

It's easy to multiply fractions, using procedures taught in class. But how many of our students can demonstrate that with an area model or number line?



$$4 \times \frac{2}{3} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$$



Regarding PISA scores in Mathematics:

In every country, the **memorizers turned out to be the lowest achievers**, and countries with high numbers of them also had the highest proportion of teens doing poorly on the PISA math assessment. Further analysis showed that memorizers were approximately half a year behind students who used relational and self-monitoring strategies. In no country were memorizers in the highest-achieving group, and in some high-achieving economies, the differences between memorizers and other students were substantial. In France and Japan, for example, pupils who combined self-monitoring and relational strategies outscored students using memorization by more than a year's worth of schooling.

Another barrier we face in triangulating evidence is our *lack of confidence in our own judgement*. If a student does a written assessment, say a test or Sask Common Math post test, it is very easy in black and white to see what they are capable of (or what they were capable of, mostly symbolically, in that particular moment, based on indications of algorithms, computation, and number answers). Judging student's oral explanations and manipulation of objects feels much more subjective to teachers. Combining black and white "product" evidence with more subjective "observation/conversation" anecdotal evidence feels less valid than our old methods of crunching numbers, finding averages, and reporting in percents. It may feel more defensible to have columns of numbers that are combined or averaged for an outcome mark. But Herbst and Davies, well known for international leadership in assessment, assure us that **a teacher's professional judgement, based on everything they know about a student and ongoing observation, is more valid and reliable than an external assessment.**

"Assess" -from the Latin word Assidere, which means *to sit beside*.



Finally, assessment promotes learning. Evaluation and grading do not.

Completing the Square: Concrete, Representational, Abstract

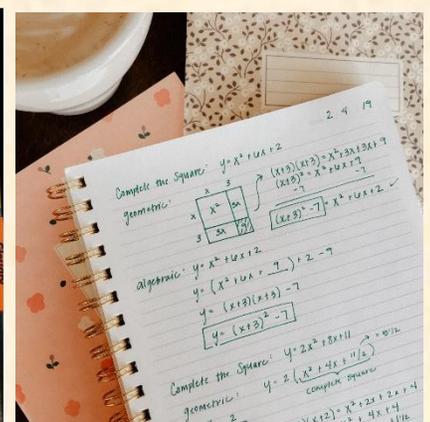
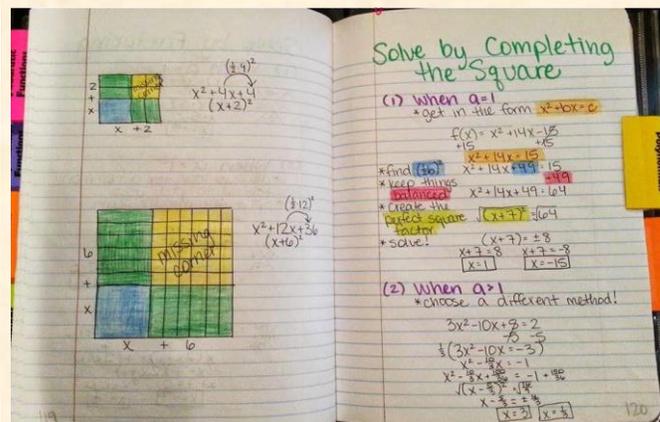
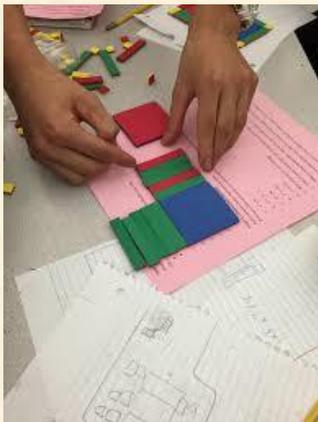


Image sources: <https://misteristhisright.com/2016/01/24/my-favorite-completing-the-square/> <https://www.pinterest.ca/aharrington2804/completing-the-square-project/> <https://medium.com/i-math/how-to-complete-the-square-8ca76a416972>

Read/view more:

<https://saskmath.ca/using-the-triangulation-of-data/>

[Assessment and the Triangulation of Data](#)

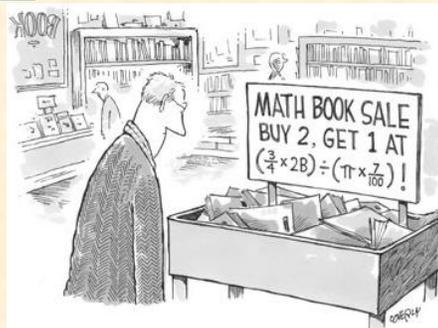
Triangulating Assessment Evidence <https://youtu.be/ByL6IMg2qPA>

How to triangulate evidence in high school math <https://youtube/isvMH2ks0hg> . Though this video talks about a specific program, "FreshGrade", the principle of triangulating and collecting evidence of growth over time is the same no matter what method you use.

Good ideas for triangulating <https://youtu.be/aY8cw7x9054> z

Podcast: How to capture observations and conversations in math class

<https://makemathmoments.com/episode143/>



Ideas for collecting evidence from observations and conversations. Best tip: Keep it simple!

MATH TALK CONVERSATION STARTERS

LET'S TALK MATH!

- TO EXPLAIN:**
 - The strategy I used was...
 - I noticed that...
- TO CLARIFY:**
 - Can you explain how/why...
 - I have a question about...
- TO AGREE:**
 - I agree with _____ because...
 - My strategy is like yours because...
 - That solution makes sense because...
- TO DISAGREE:**
 - I disagree with _____ because...
 - The solution doesn't make sense because...
- TO EXTEND:**
 - I would like to add onto...
 - Another strategy we could use is...

Mrs Winter's Bliss

Math Talk Expectations

In our learning community, we...

1. treat each other with respect
2. participate and actively listen to others.
3. give others time to think.
4. make mistakes and can revise our thinking.
5. share our ideas and ask good questions.

Mrs Winter's Bliss

Observation/Assessment Page Collection

Student Name: John R. Group/Level: Low

Name: John R. Date: Sept. 18

Skills demonstrated:

- Read numbers and in context
- All of the base
- Some of the base
- None

Observation/Assessment Steps:

- Fluently with base ten blocks

Name: John R. Date: Sept. 28

Skills demonstrated:

- Student demonstrates an ability to
- Add the given number a variety of ways
- Identify patterns in the components (tens one number goes up the other goes down)
- Find all combinations for composing a given number

Observation/Assessment Steps:

- Great work today! Easily represents number combinations to do!

Mrs Beattie's CLASSROOM

From <https://www.mrsbeattiesclassroom.com/2017/07/how-to-track-guided-math-data.html>

From <https://mrs wintersbliss.com/math-talk-in-the-classroom/>



ANECDOTAL NOTES Date _____

Concept/Skill _____

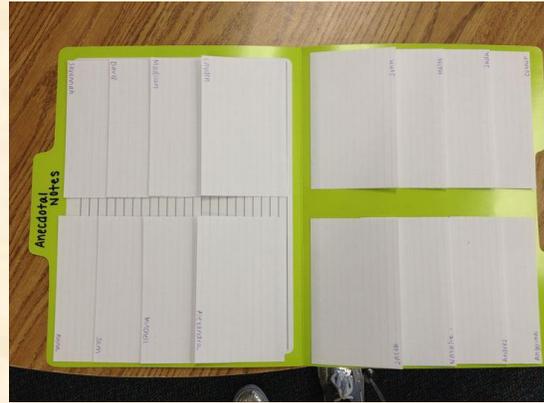
	Mastered	Progressing	Emerging
1			Needs Concept Instruction Needs Basic Skills Practice
2			
3			
4			
5			
6			
7			
8			
9			
10			
Enrichment	Follow Up	Follow Up	

Absent: _____

From <https://jenniferfindley.com/small-group-math-assessment-and-data-tracking/>



<https://thriftyinthirdgrade.com/guided-math-anecdotal-notes/>



Upcoming Math PD:

May 18 Kyle Pierce and Jon Orr—

Worksheets: Wretched or Wonderful?

How to re-work your worksheets to engage and enhance learning.

[SIGN UP FOR FREE!](#)



More information <https://makemathmoments.com/worksheet-webinar/>

What You'll Learn In This Webinar

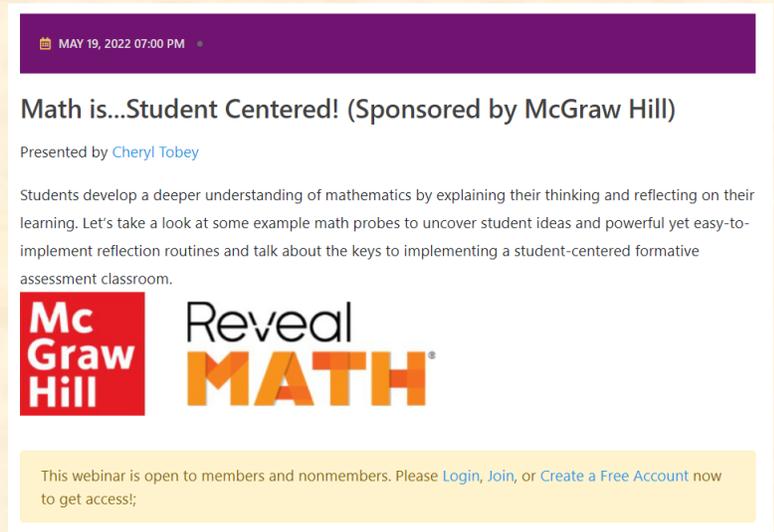
- ✔ The answer to: "when is the right time to use a worksheet?"
- ✔ The difference between a worksheet intended to "get the work done" or a worksheet intended to enhance the learning experience during the lesson;
- ✔ The types of problems, prompts or questions that should go on your worksheet and what you want to avoid;
- ✔ How to ensure your worksheet supports all levels of learners; and,
- ✔ How you can use a worksheet to enhance your problem-based lessons *without* robbing the thinking of your students.

Student Centred Math by McGraw Hill

<https://www.nctm.org/online-learning/Webinars/Details/603>

You can also check out NCTM's past webinars here

<https://www.nctm.org/online-learning/Webinars/List?status=recording>



MAY 19, 2022 07:00 PM

Math is...Student Centered! (Sponsored by McGraw Hill)

Presented by Cheryl Tobey

Students develop a deeper understanding of mathematics by explaining their thinking and reflecting on their learning. Let's take a look at some example math probes to uncover student ideas and powerful yet easy-to-implement reflection routines and talk about the keys to implementing a student-centered formative assessment classroom.

McGraw Hill **Reveal MATH**

This webinar is open to members and nonmembers. Please [Login](#), [Join](#), or [Create a Free Account](#) now to get access!;

Cool Stuff

Looking for math centres Gr K – 6? <https://www.k-5mathteachingresources.com/5th-grade-number-activities.html> has a well curated collection. Some come in bundles you need to pay for, but lots are free!

Some examples:

Place value compare <https://www.k-5mathteachingresources.com/support-files/place-value-compare.pdf>

Representing Decimals <https://www.k-5mathteachingresources.com/support-files/representing-decimals.pdf>

Interpreting remainders <https://www.k-5mathteachingresources.com/support-files/word-problems-interpreting-remainders-without-answer-key.pdf>

No end of cool stuff at Mathigon. Here is their collection of year end tasks. Take a peek around this site for other great and innovative resources!

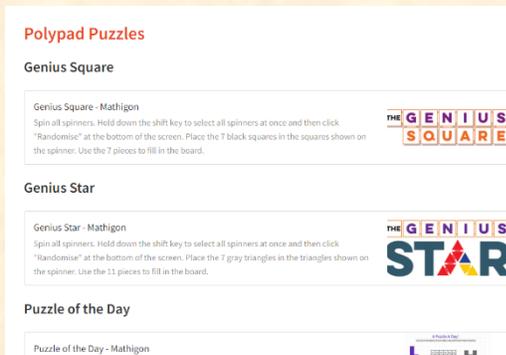


Polypad Art Contest

Submit your art, puzzles or games and win up to \$500!

Timeline of Mathematics Scavenger Hunt

The Timeline of Mathematics at mathigon.org/timeline is a wonderful place to explore some of the greatest mathematicians and some of the biggest mathematical discoveries in history. In this scavenger hunt, students search the timeline for a variety of mathematicians and artifacts.



Polypad Puzzles

Genius Square

Genius Square - Mathigon
Spin all spinners. Hold down the shift key to select all spinners at once and then click "Randomise" at the bottom of the screen. Place the 7 black squares in the squares shown on the spinner. Use the 7 pieces to fill in the board.

Genius Star

Genius Star - Mathigon
Spin all spinners. Hold down the shift key to select all spinners at once and then click "Randomise" at the bottom of the screen. Place the 7 gray triangles in the triangles shown on the spinner. Use the 11 pieces to fill in the board.

Puzzle of the Day

Puzzle of the Day - Mathigon
Use the 8 pieces to piece up your favorite current month and current day. For example...

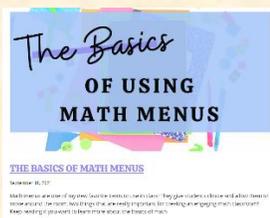
High School Teachers! Here's a collection of stuff just for secondary maths

<https://www.resourceaholic.com/2022/05/5-maths-gems-157.html> It's UK but lots of the tasks transfer

I'm enjoying this blog lately, after watching her webinar at the Math Summit

<https://mathwithsmatherson.com/blog/>

There are ideas for Math Menus as a way of structuring small group instruction in math, tips on Guided math, plus specific ideas and resources based on topics

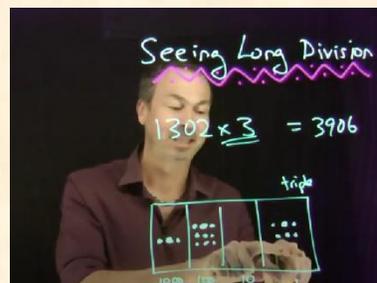


Have you seen this yet???? Exploding dots makes number operations visual. Looking for a different way to explain things? This is your place! <https://www.explodingdots.org/>

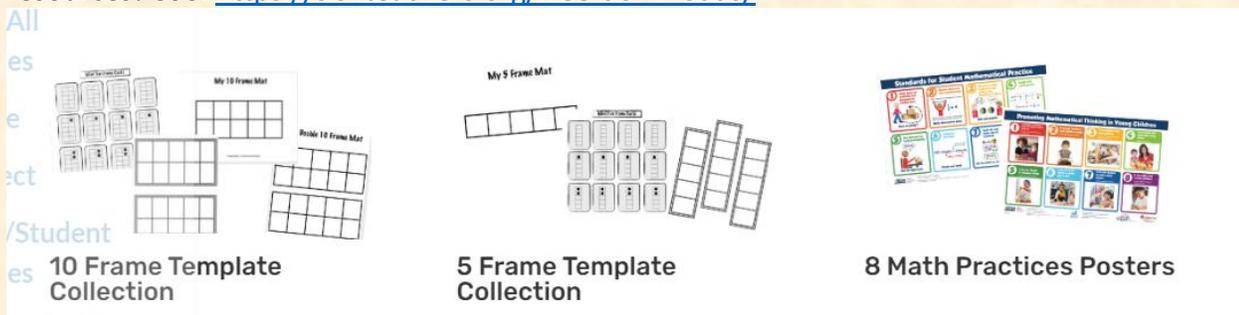
Here's what I mean <https://youtu.be/9Ea9YGN6SJM>



Strategic Intervention Solutions (SIS) FREE



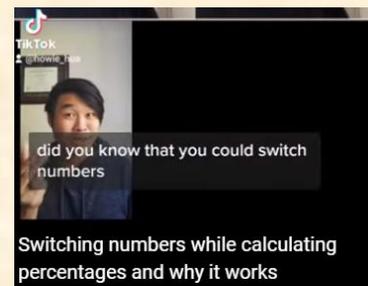
resources! Cool <https://sis4teachers.org/free-downloads/>



Neat, exit card samples <https://buildingmathematicians.wordpress.com/exit-card-samples/>



Howie Hua—I know I've mentioned him before, but if you haven't checked out his Tiktok videos (also on Youtube), have a look! He's got a great way of explaining math and making relationships visual! In this video he explains an easy way to mentally calculate percents



<https://youtube.com/shorts/N29tQAw7nBg?feature=share>

Science in math class: This should make for some good discussion! This video compares the gravity on planets in our solar system. Kids will love it!

https://www.reddit.com/r/Damnthatsinteresting/comments/t9j91e/perception_of_gravity_in_different_celestial/



(Umm, full screen the video and don't scroll around, as some comments are not classroom appropriate. Or, view it on my Twitter feed)

Problem Strings

#MathStratChat

Relational Thinking

As Close As It Gets

Find more of our favorite instructional routines below!

- I Have, You Need
- Count Around

Just added to the document "Getting Started with Number Talks" is this site, "Instructional Routines" with Pam Harris. Check out "As Close as it Gets"

<https://www.mathisfigureoutable.com/instructionalroutines>

As Close As It Gets

Use reasoning to choose the best answer.

3.219 + 5.2

a) 3,700 b) 3.7

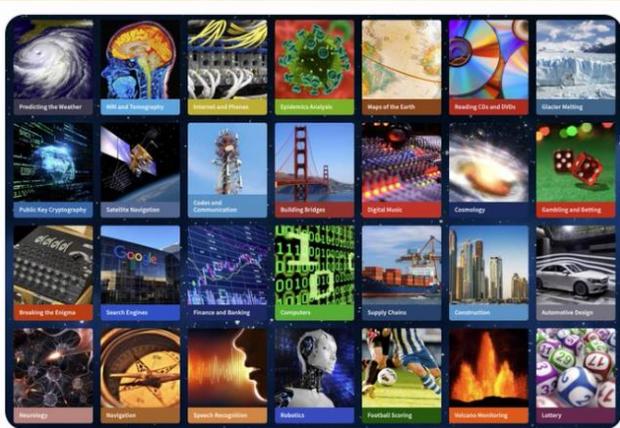
c) 8.5 d) 8

PAM HARRIS

Applications of Mathematics Mathematics has applications across so many parts of everyday life – weather prediction, bridge design, crowd control, music shuffling, and fraud detection, just to name a few! Explore

many more interesting examples at

<http://mathigon.org/applications>.



A fun challenge for a Tuesday ...and many more

<https://mathequalslove.net/>. Curated by a high school math teacher, this site has challenges for secondary students, but also great for enrichment!

2/22/22

TWOSEDAY CHALLENGE

Using exactly four twos, add arithmetical symbols between the twos to make each of the target numbers. You may use plus, minus, times, and divide symbols, as well as parentheses and brackets for grouping.

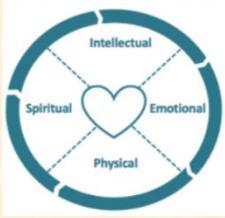
2 2 2 2	=	0
2 2 2 2	=	1
2 2 2 2	=	2
2 2 2 2	=	3
2 2 2 2	=	4
2 2 2 2	=	5
2 2 2 2	=	6
2 2 2 2	=	10
2 2 2 2	=	12

Source: 100 Numerical Games by Pierre Berloquin

M + A + T + H = Love

FREE

Indigenizing Math Instruction



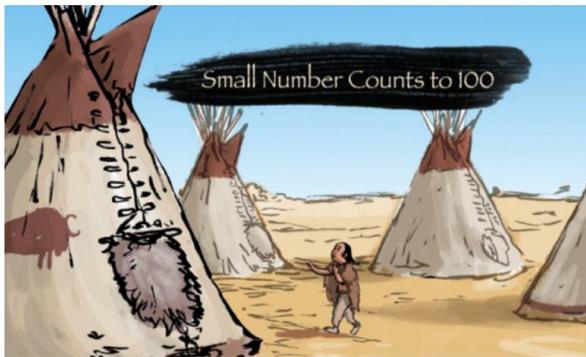
Like many changes we make, like universal design, educators are finding that instruction that incorporates Indigenous ways of knowing benefits all learners. All our students need to see themselves in our classrooms and curricula. Every math outcome Sask K-8 includes indicators that incorporate First Nations contexts. Here are some ways to learn more.

<https://saskmath.ca/indigenous-ways-of-knowing-links-for-effective-instruction/>

<https://indigenous.mathnetwork.educ.ubc.ca/>

https://www.strongnations.com/store/item_display.php?i=3458

<http://mathcentral.uregina.ca/RR/database/RR.09.00/treptau1/>



WATCH THE MOVIE!

English

Cree

Blackfoot

French

Spanish

These are the stories of Small Number—First Nations traditional style stories with Math focus. They are also available in French!

The first story, *Small Number Counts to 100* was inspired by narration from Ms. Rina Sinclair of the Siksika Nation. The story can be shown to elementary school students as a counting practice/puzzle or as a pattern recognition problem. For high school students it can be a way to introduce arithmetic progressions, modular addition, or an idea of number systems with a base different than 10.

<http://www.sfu.ca/mathcatcher/StoriesMovies.html>

Ten Formative Assessment Activities for Middle and Senior Mathematics

Formative Assessment is **assessment for learning**. Immediate feedback is important for student achievement. Remember to keep some type of data on your formative assessments. --even though they are not grades they are still evidence used to justify instructional decisions, student placement and programming, and are used as talking points for Parent/Student/Teacher conferences. Involve students in the assessment process by having them self and peer assess as well.

1. Exit Slips

Exit slips or exit cards can be used regularly to see what students have learned and understand about a lesson or unit of study. They can be given out near the end of class, and should be designed so that they can both be completed quickly and assessed quickly. Deal with exit slips before planning the next lesson so that you can respond to the data. You may need to adjust instruction, reteach, move more quickly, or fill in gaps in background knowledge. Exit slips will also reveal which students have grasped a concept and are ready to move on, and which need more support or intervention.

You can differentiate exit slips by offering questions at a variety of levels (parallel tasks) and asking students to choose one. You will be surprised which students choose more difficult ones, and which students will just do them all to really show you what they know! Bu offering

choice in the differentiation, you are ensuring that you are not prejudging student abilities, and that you are “teaching up” for all your learners.

2. **Fist to Five**

This is a quick way to check for understanding before, during or after a lesson. Ask students to indicate their level of understanding by on a 0 – 5 scale by showing five fingers if they really feel they understand the concept, 2 or three if they feel they are only starting to learn the material, zero (fist) if they feel they totally missed the concept. If students are reluctant to show their feedback they can face the front of the room and hold their hands in front of their bodies where peers can’t necessarily see the result. This is a form of self-assessment as students are communicating their understanding of their learning.

3. **Commit and Toss:**

This is an activity that works well as a preassessment, especially in areas where there may be opinions or misconceptions. Commit and toss facilitates dialogue and debate, which are the golden learning moments in math class! Even though this activity may initiate healthy argument, it also does so anonymously. Before doing this activity, you will need to establish classroom norms that promote respectful listening and honour anonymity.

Write the question on a slip of paper which is given to each student. Students need to commit to an answer. Some optional answers may be provided, or it can be open response. Students must justify their reasoning in writing.

After all students have completed the question and explanation, have them crumple their papers and toss them around the room to each other until you say stop. Once you stop the tossing, students can read the response on the sheet they are holding and present the answer and argument it contains. Students can NOT present their own paper and opinion, only that of the paper they are holding. They may offer counter explanations afterward.

In the case of an “active” class, this activity can be modified to “fold and pass”!

4. **Quick Write**

This is often used as a preassessment to tap and activate prior knowledge. Students use paper provided, notebooks, or journals to respond very quickly to a prompt. In a short span of time student should write all they know about a topic. Using a quick write as a preassessment focusses the class on the topic or learning target of the lesson. They can then share with class, share with partner, or simply hand in the quick write so the teacher can assess what students know and understand. After teaching you can ask students to return to their quick write to see what they have learned, how their repertoire of knowledge has expanded.

5. **Think-Pair-Share**

This is a very powerful instructional practice because it allows all learners to Their ideas heard. While whole class discussion and dialogue is extremely important, it is nearly impossible to stop some students from dominating those conversations. Some students are very reluctant to speak up in whole group. After giving a thinking prompt or question, ask students to think about it for 30 seconds (at least). This thinking time is important! We rarely give sufficient wait time when we question students, and research shows that with 30 seconds of uninterrupted wait time we will get more correct responses and more complete responses. After allowing some wait time, have students turn and talk to an elbow partner or table partner, or you can randomly assign a new partner! The pair should share their

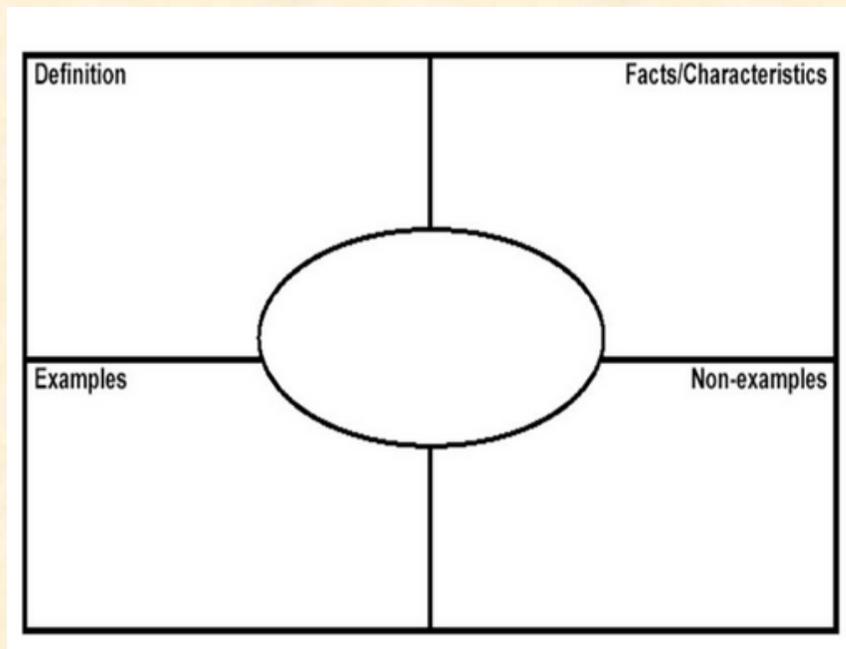
ideas, opinions, strategies, solutions, etc. Again, you must prep you class by creating norms that foster a safe and respectful learning environment.

After pairs have discussed a concept you can have them share with the class. Its often interesting to use this activity when there is a common misconception, and then see how many pairs debate their way to more correct understanding.

Think pair share is very effective for girls in math class, who are generally socialized to speak out less. It is also a good fit for first nations learners and EAL learners.

6. Frayer Model

This is a graphic organizer that can be used as a preassessment or any time during learning. It helps activate students knowledge about meaning of math concepts or words. It is very useful in generating vocabulary words or reviewing concepts. The frayer model focusses on identifying similarities and differences, which is a very effective learning strategy, as well as solidifying conceptual understanding. The frayer may help students to self-assess, because it may help them identify gaps in their knowledge.



7. Two Stars and a Wish

This is a form of comments-only marking, which is one of the most effective formative assessment practices. Giving students specific, descriptive feedback has a significant effect size as an instructional practice. Two stars and a wish helps students clarify what they are doing well and what they need to work on. It is more significant than merely saying “well done” or “good job”, and it encourages students to take action on areas that need improvement.

The two “stars” are two points or comments that point out where a student is having success at a task or assignment, and the “wish” is an area that needs improvement. Some people advocate for “sandwiching” the wish (constructive criticism) between the two stars (accolades for work well done). This is a good model when teaching peer assessment. Students can give feedback to each other, but critical comments should always be sandwiched between two bits of positive feedback that is very specific.

8. Whiteboarding

Just like wakeboarding, snowboarding, or longboarding, this is tons of fun. Ok maybe not quite as much fun, but research does show that when students can use whiteboards to give feedback they are more motivated. This is simply a very visual way for students to show their thinking.

Whiteboards can be used in collaborative learning environments for groups to demonstrate ideas and strategies, or as a method of displaying quick answers so teachers can easily see who's computing correctly and who needs more help. As students present their whiteboards to the class, the teacher can give feedback, suggestions, or clarify the concept to students who need it.

"Whiteboard presentations provide an opportunity for teachers to give feedback to students on their communication skills, such as how to share ideas so that others can understand their reasoning, how to listen carefully to critique others' ideas, how to look for commonalities in thinking, and how to engage in mathematical argumentation in a constructive way, including coming to consensus when there are differences in opinions and ideas." (Keeley and Tobey, 2011).

There are many variations on whiteboarding, such as using document cameras or scrap paper, cell phones, etc. Economy whiteboards can be made out of disposable paper plates, and mini gloves are sometimes used as handy erasers.

9. Three-Two-One

This is an activity that can be varied to collect whatever data a teacher is interested in. It is a quick write, can be used as an exit slip or journal entry. An example that collects data on learning is "tell me three things about this unit you understand well, two things you need to work on, and one question you still have". If you need to collect data on classroom environment you may ask: State three things that help you to learn mathematics, two things that are challenging to you and one wish". You can customize the prompts to anything you like, but they should be meaningful and promote reflection. Using assessments like 3-2-1 not only provide valuable information about instruction, learning, and classroom dynamics, but also help teach students to monitor and self-reflect.

10. K-W-L

What do you already know about this topic? What do you want to learn about the topic? And later we will review what you have learned about the topic. Students fill in a K-W-L chart as a preassessment, which helps teachers know what information students already have, and where there may be gaps in background knowledge. This activity also helps students monitor their own learning and reflect on learning by revisiting their answers after learning has taken place. This can be motivational as well, because it demonstrates what progress students have made by showing what has been learned.

Topic _____		date _____
<u>K</u> now What I think I know	<u>W</u> onder What I want to know	<u>L</u> earn What I learned

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Reference:

Keeley, P., & Tobey, C. (2011).
Mathematics Formative
Assessment. National Council of
Teachers of Mathematics, Corwin.
Thousand Oaks, CA.