Responsive Instruction: Classroom Level Supports

Module 3: INSTRUCTION IN MATHEMATICS

Effective Instructional Practices

2020

Disclaimer: Please note the Ministry of Education is not endorsing any professional resources or specific products found in this document. The resources are intended to provide additional information about the topics addressed in the document. The external links may contain advertisements and other content the ministry is not aware of nor endorses.

Preface

Saskatchewan is guided by a Student First approach which puts the student front and centre by focusing on each student's strengths, abilities, interests, and needs. To create intentional and responsive educational experiences in classrooms and in the school involves the entire school community, including students and families.

The Saskatchewan Ministry of Education promotes the belief, attitude and approach of inclusion for meeting student needs. A strong commitment to inclusive education by the school community is demonstrated by inclusive educational practices that are part of the everyday school experiences of the student. Environments where students feel safe, accepted, respected and confident to engage in learning are essential to student success.

Supporting students through a needs-based service delivery model promotes the success of all students, including those who have learning needs that require supports to optimize learning opportunities. The needs-based model is a Student First strengths-based approach in which responsive instruction, interventions and supports are identified, planned and provided to meet student needs at the school and classroom level**,** through targeted and/or group approaches, and at an intensive individual level. The needs-based model recognizes that a student's needs change over time and that individualized, flexible and responsive supports are required.

Instruction in Mathematics: Effective Instructional Practices is part of a series of modules that provide examples of responsive instruction to support student learning. **This module focuses on effective instructional practices that support the learning needs of students and enhance mathematical thinking.**

Supporting documents and resources can be found throughout the module by clicking on the underlined term, phrase or title.

Introduction

Instruction in mathematics builds upon students' prior learnings and continuously develops their number sense, spatial sense, logical thinking, and understanding of mathematics as students move from one grade to the next, and beyond Grade 12. These continuing learnings prepare students to be confident, flexible and capable users of their mathematical knowledge in new contexts (Ministry of Education, 2008).

However, some students experience ongoing difficulty acquiring even the most basic mathematical skills. These students may exhibit some or all of the following behaviours:

- early difficulties in estimating quantities, subitizing (the ability to see a small amount of objects and know how many there are without counting) and associating small quantities of items with printed numerals;
- persistent and continuing difficulty with understanding basic number facts and apparent inability to develop efficient memory strategies on their own;
- inconsistency in attending to operational signs and calculating, as well as difficulty sequencing the steps in complex operations, despite having an overall grasp of math concepts;
- difficulties with the language aspects of mathematics, resulting in confusion with terminology;
- difficulty following verbal explanations;
- difficulties in visual-spatial organization, which may result in confused arrangements of numerals and signs on the page, poor 'number sense', difficulty with interpreting pictorial representations and/or difficulty understanding math concepts; and,
- working memory difficulties that may interfere with multi-step problem-solving activities (adapted from LD@school, 2019).

Teachers need to be familiar with how their students learn in order to determine appropriate assessment and instructional strategies, and make appropriate adaptations to support student learning. Learning mathematics efficiently requires many different skills, including logical and strategic thinking; fact retrieval from memory; visualization; and the ability to order, organize, sequence and focus on a problem.

To support student learning, teachers need to be aware of their students' strengths (i.e., the skills they have) and challenges (i.e., the skills they need), learning styles, preferences and interests to differentiate effectively in the classroom. They also need to make instructional choices to support the diverse learning needs of students and enhance mathematical thinking.

Effective Instructional Practices

The purpose of mathematical learning is for students to be able to transfer mathematical skills and knowledge into real-world situations. In order to do this, teachers need to understand the difference between procedural and conceptual knowledge and consider this difference when planning (Pennsylvania Department of Education, 2015).

> Designing effective instruction in mathematics involves balancing understanding of mathematical concepts with procedural fluency. Effective instruction involves intentional approaches, strategies, and learning activities based on mathematical and pedagogical knowledge and understanding of student mathematical development.
>
> Ontario Ministry of Education, 2015

Procedural knowledge is developed when an individual learns certain skills or the steps required to complete a task. Students can use procedural knowledge to find a correct solution or answer without really understanding how or why the solution was reached.

Conceptual knowledge is the acquisition or development of a deeper, more meaningful understanding of mathematical relationships and content. Students may use procedures or algorithms to complete a task but can demonstrate conceptual knowledge by explaining how they reached an answer, defend their solutions, demonstrate their learnings in multiple ways and make connections to new learnings.

Example of Procedural Knowledge: Communicating in a symbolic manner (e.g., *three-halves divided by one-half*) does not connect to the meaning of division of fractions. It may simply involve repeating a memorized rule (such as "invert and multiply"), without building understanding that division represents how many groups/sets/lengths are in a given amount.

$$\frac{3}{2} \div \frac{1}{2} = \frac{3}{2} \times \frac{2}{1} = \frac{6}{2} = 3$$

Example of Conceptual Knowledge: A student with conceptual knowledge would understand that the above problem is asking how many groups of $\frac{1}{2}$ are in $\frac{3}{2}$ and be able to share their understanding in different ways such as symbolically, pictorially and demonstrating with manipulatives.

Instruction should include both procedural and conceptual approaches as these two types of knowledge usually develop iteratively, one knowledge influences the other (Kadijevich, 2018). Mathematical literacy involves learning procedures, choosing appropriate procedures and understanding the reasons why they work.

Jayanthi, Gersten, and Baker (2008) recommend several instructional practices that have been proven to support a wide range of learning needs during mathematic instruction. These practices can be implemented when students are acquiring procedural (mathematic skills) and conceptual knowledge (understandings). Their recommendations are based on the findings from a meta-analysis report on the topic of teaching mathematics to students with learning disabilities (Gersten, Chard, Jayanthi, Baker, Morphy & Flojo, 2008 as cited in Jayanthi et al., 2008) and recommendations from *The Final Report of the National Mathematics Advisory Panel* (National Mathematics Advisory Panel, 2008 as cited in Jayanthi et al., 2008).

Most of the recommendations are not new, but rather, are practices that have stood the test of time and have surfaced repeatedly in the literature as being effective supports for student learning in mathematics. Based on their findings, when planning for mathematical instruction teachers should consider:

- using formative assessment data to inform instruction;
- using explicit instruction;
- having students verbalize approaches and solutions to a mathematic problem;
- teaching students to visually represent the information in the mathematic problem;
- teaching students to solve problems using multiple heuristic strategies; and,
- providing peer-assisted instruction opportunities to students.

Formative Assessment

Ongoing formative assessment refers to a wide variety of methods teachers use to conduct in-process evaluations of students' progress in mathematics. Formative assessments involve the use of information about student progress to support, improve and inform student learning and instructional practices. Teachers use the information from formative assessment techniques to differentiate instruction and provide immediate feedback to students to enhance their learning (Ministry of Education, 2009).

Formative assessment takes place during teaching to make adjustments to the teaching process (Van de Walle et al., 2011). It occurs when teachers plan a process of regularly checking student's understanding during instructional activities. Teachers use the results and evidence collected to improve instruction for individual students, small groups and/or the whole class.

Research suggests that effective formative assessment include:

- clarifying, sharing, and understanding what students are expected to know;
- creating effective classroom discussions, questions, activities and tasks that offer the right type of evidence of how students are progressing to the espoused learning goals;
- providing feedback that moves learning forward;
- encouraging students to take ownership of their own learning; and,
- using students as learning resources for one another (National Council of Teachers of Mathematics, 2007).

Explicit Instruction

Explicit instruction is a very practical yet effective model of instruction (Alphonse, J. A. & Leblanc, R., 2014). Explicit Instruction resembles the Gradual Release Model or the I Do, We Do, You Do or [You Do, We Do, I Do] model of teaching. Both models require student participation, engagement and collaboration.

Explicit instruction can be used at all grade levels, with an entire class or during small group or individual instruction and across all content areas and with all students. It is most effective after formative assessment has taken place and the teacher has an understanding of how to respond to students or student in a manner that moves learning forward. Explicit instruction has been found to be especially successful when a student has problems with a specific or isolated skill (Steedly, K., Dragoo, K., Arafeh. S, & Luke, S., 2008). The elements of explicit instruction include:

- focusing instruction on critical content;
- sequencing skills logically;
- breaking down complex skills and strategies into smaller instructional units;
- designing organized and focused lessons;
- beginning lessons with a clear student-friendly statement of the lesson's outcome and expectations;
- providing step-by-step demonstrations;
- using clear and concise language;
- providing an adequate range of examples (when to use the skill) and non-examples (when not to use the skill);
- providing guided and supported practice;
- requiring frequent student responses;
- monitoring student performance;
- providing immediate affirmative and corrective feedback;
- delivering the lesson at a pace that holds students' attention;
- helping students organize knowledge; and,
- providing [purposeful practice] (practising the same skill repeatedly e.g., subtraction) and cumulative practice (practising both previously and newly acquired skills e.g., addition and subtraction).

(adapted from Archer & Hughes, 2011)

Explicit instruction has three components or steps: modeling (I do), guided or directed practice (we do), and independent (you do). The order of the steps is determined by the purpose of the lesson (e.g., a discovery approach may begin the lesson with "You do") and student need.

Explicit instruction focuses on teaching students how to learn by modeling what efficient learners do to understand and learn new material or skills.

Explicit instruction includes teaching components such as:

- presenting multiple examples of the problem;
- thinking the specific steps aloud during modeling;
- clear modeling of the solution specific to the problem; and,
- providing immediate corrective feedback to the students on their accuracy.

When teaching a new procedure or concept, teachers need to focus on sequencing instructional examples and selecting a range of examples of a problem when planning mathematical instructions (Jayanthi et al., 2008). Multiple examples can be presented in a specific sequence or pattern such as concrete to abstract, easy to hard, or simple to complex. Sequencing examples are important during early acquisition of new skills when scaffolding is needed for student mastery and success. Providing a range of examples may support students when transferring learned skills to new situations and problems (e.g., using the symbol 1/2, the word “half”, the word “one-half”).

“I do” - Teachers model how to find solutions to specific problems while talking through the specific steps aloud. The goal of modeling is for the teacher to explicitly state what they are doing, why they are doing it and how they are doing it as students observe.

“We do” - Guided or directed practice provides a way for students to succeed in achieving outcomes of the lesson (Alphonse, J. A. & Leblanc, R., 2014). Students receive immediate feedback, guidance and support as they work through tasks. Students are provided with the opportunity to verify and adjust their thinking while learning new skills. Teachers provide guided practice to individuals or small groups of students while they circulate the room.

“You do” - During this step, students apply what they have learned as they work independently or teachers can assign a task at the beginning of a lesson and observe what students can do independently. Teachers can use this step as a source of assessment to evaluate what students already know or have learned and use this information to inform their instruction.

Encourage Students to use Think-Aloud Strategies

Think-Aloud strategies encourage students to verbalize approaches and solutions when solving math problems. Verbalizing helps students reflect upon and clarify the problem they are trying to solve and focus on solving it one step at a time. Thinking aloud requires students to talk through the details of the problem, the decisions they have made as they try to solve the problem and the reasoning behind those decisions (Zorfass, J., Gray, T. & *PowerUp What*

Works, 2015). By verbalizing their inner speech (talking to themselves) students think their way through a problem. By listening in as students work, teachers can quickly assess students' strengths and weaknesses and correct any misunderstandings the student might have regarding the task.

For students who are impulsive or have difficulty attending, verbalizing steps while problem solving may help them slow down, giving them time to focus on the key parts of the problem. This helps them to more fully comprehend the problem before they try to solve it and may help to self-regulate (Jayanthi et al., 2008).

Visual Representation

Visual representations (drawings, graphic representations) are used by teachers to explain and clarify problems and by students to understand and simplify problems. When used systematically, visuals have a positive impact on students' mathematic performance (Jayanthi et al., 2008). Representing information visually, like any other skill, must be taught and practised. A common strategy used to teach students how to use visual representations is the concrete-representational-abstract approach (CRA).

CRA works well with individual students, in small groups or with an entire class at both the elementary or high school level. CRA is a three-part instructional strategy in which the teacher first uses *concrete* materials (e.g., base-ten blocks or geometric figures) to model the mathematical concept to be learned, then demonstrates the concept in *representational* terms (e.g., drawing pictures), and finally in *abstract* or symbolic terms (i.e., numbers, notation, or mathematical symbols). Some students may need to continue using manipulatives in the representational and abstract stages, as a way of demonstrating understanding.

Heuristic Strategies

Heuristics are simple rules and mental shortcuts that have been created based on past experiences. These types of strategies are best used with students who understand the mathematic concept, but have trouble remembering the steps in completing a problem. Teachers and students can create heuristics to reduce the number of mental operations (or information-processing steps) taken to solve a problem. This strategy exemplifies a generic approach for solving a problem, moves students away from memorizing algorithms and develops a deeper understanding of math problems.

Examples of heuristic strategies are found below.

Guess-Check-Revise – in this strategy students draw from their experience and make a guess about the solution. They then check to see if their guess solves the problem. If the guess does not solve the problem the student revises their guess and starts over.

Make a Systematic or Organized List – students construct an organized list to solve math problems.

Try a Simpler Form – students modify or simplify the quantities in a problem so that the resulting task is easier to understand and analyze. Solving the easier problem may lead the student to insights that can be used to solve the original, more complex task.

Working Backwards – students start with the answer or result of a problem and apply operations in reverse order until they find the starting number.

These types of strategies have been found to encourage students to take risks and improve student engagement during problem solving activities (Novotná, J. et al., 2014).

It is important that problem solving strategies, including heuristics, do not become "proceduralized" by telling students the strategy they should choose (Van de Walle et al, 2011). Students need opportunities to explore and discover which problem solving strategies work best with different types of problems.

Example: A teacher poses a problem to the class that lends itself to a targeted strategy (e.g., making an organized list). The teacher encourages the class to solve the problem as they like. When discussing the results, the teacher highlights examples of work that used an organized list and poses questions to the class as to whether this strategy was an efficient choice. The teacher then deepens the conversation by having students discuss what strategy they would choose next time for this type of problem.
If the targeted strategy was not chosen, the teacher would challenge the class to try the targeted strategy. The class would then discuss which strategy, their first choice or an organized list, was more efficient for this type of problem.

Peer-Assisted Instruction

Peer-assisted instruction or peer tutoring is an instructional strategy that consists of pairing students to learn or practise a task. Peer tutoring is a strategy that can be used with students at all grade levels. When strategically planned for and implemented, peer tutoring can have a positive influence on student motivation and academic achievement for both tutors and tutees (Bender, 2008, Topping, 2005). Peer-assisted instructional strategies provide students with increased opportunities to respond and receive immediate positive feedback, as well as increasing time on-task.

When implementing class-wide peer tutoring strategies every student should have opportunities as both the tutor and tutee. It is common for the teacher to have students switch roles partway through, so the tutor becomes the tutee. Explaining a concept to another person helps extend one's own learning, providing both students the opportunity to better understand the material being studied.

Successful implementation necessitates training all students in the process and roles of peer tutoring. This training should describe how both tutors and tutees benefit from peer tutoring, the role of the tutors and tutees, and how to ask for and offer help when working with a partner.

Conclusion

Instructional practices are often intertwined and are chosen and implemented based on the purpose of the lesson and the learning needs of the students. Teachers need to understand how their students learn, have the ability to explain why the instructional approach or strategy is the best one for a particular purpose and adapt instructional practices to meet the needs of all students (see Responsive Instruction: Targeted/group Approaches: *Module 2 - Targeting Mathematics Instruction - Knowing our Learners*).

Archer and Hughes (2011) state that there is no best way to teach mathematics. Instead, teachers should use a repertoire of instructional strategies that are responsive to student learning interests, preferences, strengths and assessed needs (Ontario Ministry of Education, 2015). All students benefit from learning through a variety of approaches, strategies and resources. Mathematics instruction should reflect what works best for students to learn.

Reflective questioning provides educators with the opportunity to think deeply about their educational ideologies and practices and how they impact students.

Effective Instruction Practices Reflective Questions and Consideration

• Do I know my students' strengths, interests and needs?
• What kind of data should I be gathering?
  ○ How should I use data to inform my instruction?
  ○ Am I using a variety of on-going formative assessments (e.g., observation, work samples, interviews)?
  ○ Have I consulted with the student support services teachers regarding different ways to assess?
• Do I circulate when students are working individually and in small groups? Am I collecting data when I meet with students individually or in small groups?
• Does my instructional plan consider the individual differences in my classroom?

- Does my instruction match the strengths and needs of my students?
- Am I connecting math concepts to other math concepts (e.g., multiplication and area) and to real-life scenarios when appropriate?
- Do I provide opportunities for my students to create and solve math problems?
  - Do I encourage my students to discuss their solutions to a math problem?
- Do I encourage my students to think mathematically (i.e., not just following learned procedures or steps)?
- Is my instruction moving students forward?
  - Am I building on what they already know?
- Do I provide opportunities for my students to learn from each other?

Want to Learn More?

Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching.* San Francisco, CA: A Wiley Brand.

Witzel, B.S. & Little, M.E. (2016). *Teaching elementary mathematics to struggling learners.* New York, NY: The Gilford Press.

Harris, K. R. & Meltzer, L. (2015). *The power of peers in the classroom: Enhancing learning and social skills.* New York, NY: The Gilford Press.

Helpful Ministry Documents and Resources:

- Actualizing a Needs-Based Model
- The Adaptive Dimension for Saskatchewan K-12 Students
- EAL, Immigration and Languages
- ESSP Reading, Writing and Math Supports: Math
- Inclusive Education
- Saskatchewan Curriculum
- Stewart Resources Centre
- Supporting All Learners

Bibliography

Archer, A. L. & Hughes, C. A. (2011). *Explicit Instruction: Effective and efficient teaching.* New York, NY: The Gilford Press.

Alphonse, J. A. & Leblanc, R. (2014). *Explicit instruction: A teaching strategy in reading, writing, and mathematics for students with learning disabilities.* Retrieved from https://www.ldatschool.ca/explicit-instruction-a-teaching-strategy-in-reading-writing-and-mathematics-for-students-with-learning-disabilities/.

Bender, W. (2008). *Differentiating instruction for students with learning disabilities: Best teaching practices for general and special educators*. Thousand Oaks, CA: Corwin Press.

Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching.* San Francisco, CA: A Wiley Brand.

Harris, K. R. & Meltzer, L. (2015). *The power of peers in the classroom: Enhancing learning and social skills.* New York, NY: The Gilford Press.

Jayanthi, M., Russell, G. & Baker, S. (2008). *Mathematics instruction for students with learning disabilities or difficulty learning math: A guide for teachers.* Retrieved from: https://www.academia.edu/26759365/Mathematics_Instruction_for_Students_with_Learning_Disabilities_or_Difficulty_Learning_Mathematics_A_Guide_for_Teachers.

Kadijevich, M. D. (2018). Relating procedural and conceptual knowledge. *The Teaching of Mathematics.* 21, (1), 15-28. Retrieved from http://elib.mi.sanu.ac.rs/files/journals/tm/40/tmn40p15-28.pdf.

LD@school – Learning Disabilities Association of Ontario (2019) Website. Retrieved from https://www.ldatschool.ca/.

National Council of Teachers of Mathematics. (2007). *Five "key strategies" for effective formative assessment.* Retrieved from: https://www.nctm.org/uploadedFiles/Research_and_Advocacy/research_brief_and_clips/Research_brief_04_-_Five_Key%20Strategies.pdf.

Novotná, J., Eisenmann, P., Přibyl, J., Ondrušová, J. & Břehovsky, J. (2014). Problem solving in school mathematics based on heuristic strategies, *Journal on Efficiency and Responsibility in Education and Science*. 7, (1), 1-6, online ISSN 1803-1617, printed ISSN 2336-2375, doi: 10.7160/eriesj.2013.070101.

Ontario Ministry of Education. (2015). *Knowing and responding to learners in mathematics.* Retrieved from: http://www.edugains.ca/resourcesDI/KnowingAndRespondingToLearners/KnowingAndRespondingToLearners_Math_AODA.pdf.

Pennsylvania Department of Education. (2015*). Essential practices for effective mathematics instruction.* Retrieved from: https://www.pattan.net/getmedia/5680fa84-b6c0-42bf-b670-688a5d8c87d2/TDR_Effect_Math0215.

Saskatchewan Ministry of Education. (2015). *Actualizing a needs-based model.* Regina, SK: Author.

Saskatchewan Ministry of Education. (2017). *The adaptive dimension for Saskatchewan K-12 students.* Regina, SK: Author.

Saskatchewan Ministry of Education. (2017). *Inclusive education*. Regina: SK: Author.

Saskatchewan Ministry of Education and Saskatchewan Professional Development Unit. (2011). *Powerful instructional practices: A resource for teachers and administrators*. Retrieved from https://www.stf.sk.ca/sites/default/files/powerful_instructional_practices_2011.pdf.

Steedly, K., Dragoo, K., Arafeh. S. & Luke, S. (2008). Effective Mathematics Instruction. *Evidence for education.* 3(1), 1-12. Retrieved from: http://www.parentcenterhub.org/wp-content/uploads/repo_items/eemath.pdf.

The Access Center. (n.d.) *Using peer tutoring for math*. Retrieved from: http://165.139.150.129/intervention/Peer%20Tutoring%20for%20Math.pdf.

Topping, K. (2005). *Trends in peer learning.* Educational Psychology. 25, (6), 631-645.

Van de Walle, J.A., Folk, S., Karp, K. S., & Bay-Williams, J.M. (2011). *Elementary and middle school mathematics: Teaching developmentally.* Toronto, ON: Pearson Canada.

Wiliam, D. (2007). *Five "key strategies" for effective formative assessment.* National Council of Teachers of Mathematics: Research Brief. Retrieved from https://www.nctm.org/uploadedFiles/Research_and_Advocacy/research_brief_and_clips/Research_brief_04_-_Five_Key%20Strategies.pdf.

Witzel, B.S. & Little, M.E. (2016). *Teaching elementary mathematics to struggling Learners.* New York, NY: The Gilford Press.

Zorfass, J., Gray, T. & PowerUp What Works. (2015). *Thinking aloud in mathematics.* Retrieved from: http://www.ldonline.org/article/63842/.