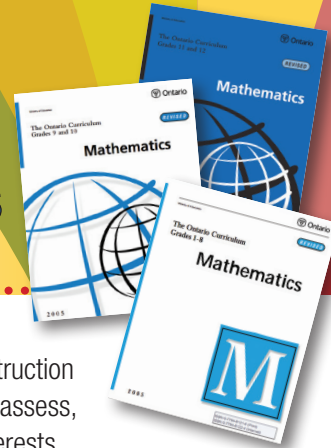




Knowing and responding to learners in Mathematics



This booklet provides an “at a glance” look at effective and differentiated instruction in Mathematics. It includes classroom scenarios that describe how teachers assess, plan and adapt their instruction to determine and address their students’ interests, learning needs and preferences.

Differentiated instruction (DI) is adapting instruction and assessment in response to differing student interests, learning preferences, and readiness in order to promote growth in learning.

Differentiated instruction may be planned prior to working with students in classrooms and also happens in the moment – as teachers adjust their instruction in ‘real time’ to respond to unanticipated strengths and needs surfacing from assessment (Parsons, Dodman and Burrowbridge, 2014).

“Mrs. Vyas always takes the time to get a better understanding of where we are so she can get us to where we need to be – like understanding *everything*.”

(Grade 9 Student)

Why use a differentiated approach to teaching and learning?

Equity of Opportunity for Learning

Ontario’s diversity is one of the province’s greatest assets. Our schools need to be places where everyone can succeed in a culture of high expectations. By ensuring equity of opportunity for learning in our education system, we can help all students achieve excellence (*Achieving Excellence, A Renewed Vision for Education in Ontario*, 2014).

The Adolescent Learner

Adolescents are in varied stages of development ... physical, emotional, cognitive and social, as they move from childhood to adulthood. Some will reach milestones at an early age; others may take more time or not reach certain milestones at all (*Stepping Stones*, 2012). In the intermediate grades, students are required to meet increased demands (e.g., vocabulary acquisition, reading skills, accessing, managing and evaluating large amounts of information) in a variety of disciplines. The range of literacy skills in a classroom often spans several grade levels. Supporting the learning of young people requires a clear understanding of the learner as well as their context for learning.

Educators play a key role in designing learning experiences that are responsive to the student’s development, strengths and needs.

Teachers who effectively differentiate:

- consistently assess student progress in multiple ways
- build extensive knowledge about how students learn and effective pedagogy
- reflect critically on their practice

(Parsons, Dodman and Burrowbridge, 2014, p. 41)

Mathematics Scenarios

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Effective and Differentiated Instruction in Mathematics

What is essential to effective instruction in Mathematics?

Designing effective instruction in mathematics involves balancing understanding of mathematical concepts with procedural fluency. Effective instruction involves intentional approaches, strategies, and learning activities based on mathematical and pedagogical knowledge and understanding of student



The seven mathematical processes describe the actions of doing mathematics. They support the acquisition and use of mathematical knowledge and skills.

(TIPS4RM: Mathematical Processes, 2009)

In a responsive, engaging and welcoming learning environment learners:

- have agreed upon ways of collaborating
- know what is expected of them
- respect and value each other's differences and work productively together
- have opportunities to work individually or in small groups
- make choices based on strengths and needs
- have flexible and reconfigurable spaces for their learning

See also: *Adolescent Literacy Guide*, 2012
DI Educator's Package, 2010
The Third Teacher, 2012



mathematical development. Using assessment to inform instruction is essential to a precise, timely and differentiated response that addresses the diversity of student learning needs.

Elements of effective mathematics instruction include:

- relevant and engaging tasks, including parallel tasks and open questions
- a variety of representations of the mathematics (concrete, pictorial, numerical and algebraic)
- access to mathematics learning tools and technology
- frequent and varied assessment of student understanding

Why is differentiation important for student learning in mathematics?

Student readiness, interests and learning preferences vary greatly within any mathematics classroom. Students will differ in their knowledge and understanding of mathematical concepts and in their use of mathematical skills such as mental math and estimation. Students also vary in their application of the mathematical processes:

- solving problems in new situations
- reasoning skills including proportional reasoning, algebraic reasoning, and spatial reasoning
- reflecting on and monitoring one's thinking
- selecting and using a variety of learning tools and computational strategies
- connecting mathematics to real life and to other mathematical ideas
- representing mathematical ideas and relationships concretely, pictorially, numerically, and algebraically
- communicating mathematical thinking orally, visually, and in writing, using everyday language and mathematical vocabulary

Responding to differences in readiness helps students feel capable and increases their motivation to learn. Addressing student interests and learning preferences (e.g., through flexible grouping and providing choice) provides relevance and autonomy – factors key to student engagement (Willms, J. D., & Friesen, S, 2012; Marzano, R.J., and Pickering, D.J., 2010).

The Mathematics Scenarios

The teachers in the following scenarios have a deep understanding of their discipline and the curriculum for the subjects they teach.

They have attended to each of the components in the Complexity of Learning and Teaching diagram by:

- establishing safe, engaging and inclusive learning environments that address the developmental needs of adolescents: affirmation, relationships, challenge, contribution, power and autonomy, purpose, and voice (*Adolescent Literacy Guide*, 2012, p.10)
- designing learning experiences that focus and engage their learners
- selecting appropriate instructional strategies that help students meet their learning goals.

Complexity of Learning and Teaching



(see page 8)

The scenarios illustrate how the teachers assess to understand the learning needs of their students, use this information to shape instruction and reflect on their practice.

The scenarios show how the teachers not only carefully plan instruction to differentiate for the variety of learners in their classroom but also adapt to meet specific, perhaps unanticipated, needs that arise during instruction. In each example there is a clear learning goal and an evident plan for differentiated instruction based on assessments of student interests, learning preferences and/or readiness. Technology is used to support and enhance differentiation. Each scenario incorporates some or all of the key features of differentiated instruction.

Key features of DI

Flexible Learning Groups

- vary over time based on student readiness, interests and learning preferences

Choice

- provides personalized opportunities for students to take ownership of their learning

Respectful Tasks

- promote high expectations; are equally engaging yet assessed using the same criteria

Shared responsibility for learning

- develops when students are supported in becoming autonomous, self-assessing learners

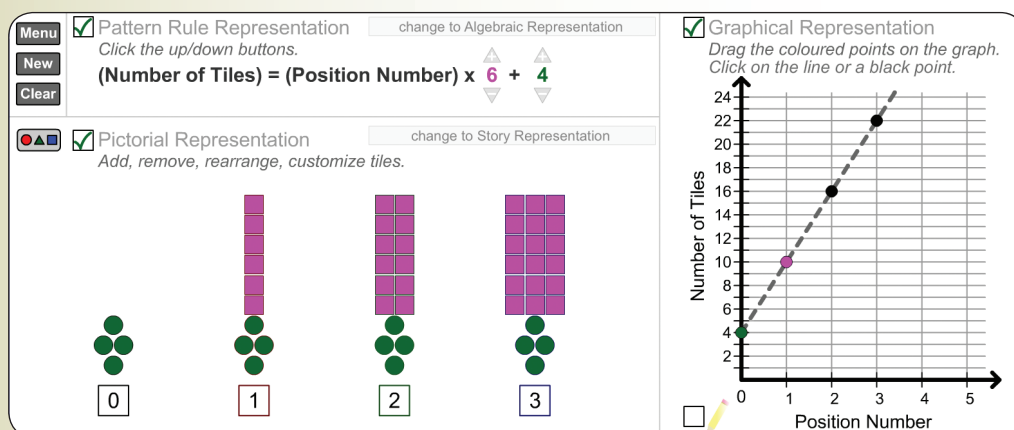
(Differentiated Instruction Educator's Package, 2010)



Grade 7 Mathematics

Helping Students Develop Algebraic Thinking Through Multiple Representations (Ms. Willow)

Ms. Willow's grade 7 class is learning to represent linear growing patterns using concrete or pictorial models, pattern rules and graphical representations. Yesterday, students used CLIPS (*Critical Learning Instructional Paths Supports*) to explore linear growing patterns (select 'Linear Growing Patterns-Representing' at <http://oame.on.ca/mathies/activities.html>) and to interact with dynamic representations. Ms. Willow uses CLIPS as a blended learning resource because it allows students to construct understanding at their own pace and to replay scenes within a clip or revisit entire clips as needed. These features help to address the wide range of learning needs within her class. Ms. Willow asks her students to self-assess their understanding of linear growing patterns on an exit card using the 'traffic light' prompts (see margin) and to indicate the type of representation with which they feel most confident.



CLIPS provides multiple opportunities for practice and enrichment with immediate levelled corrective feedback

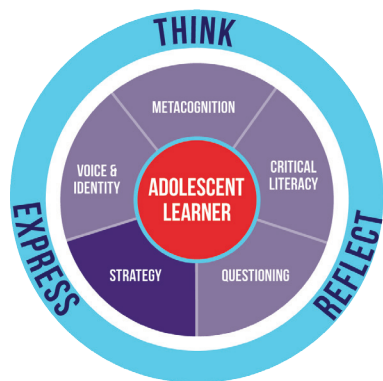
Traffic Light

RED: Stop – “I am feeling confused and need extra help”,
YELLOW: Proceed with Caution – “I think I understand but may need help or more practice”, or
GREEN: Go – “I understand and feel confident.”

For another example of the traffic light strategy see St Mary's SS video: http://www.edugains.ca/newsite/di/dvd_secondary_success.html



See www.mathies.ca for interactive, electronic supports for mathematics for Ontario K-12 students and parents



Strategy is a component of adolescent literacy. Teachers support students in strategy use by modelling subject-specific processes and explaining how they reflect thinking in the subject.

(Adolescent Literacy Guide, 2012)



Based on the information gathered from the exit cards and her observations, Ms. Willow designs tasks for three centres (Graphical, Pictorial/Concrete and Pattern Rule) to help students make connections among three ways of representing linear growing patterns. Tasks at each centre have the same learning goal but differ in the type of representation that initiates the task. For example, if the initial representation is a graph, then the task may be to build the pattern with concrete materials such as linking cubes, coloured tiles, pattern blocks, or toothpicks to determine the pattern rule.

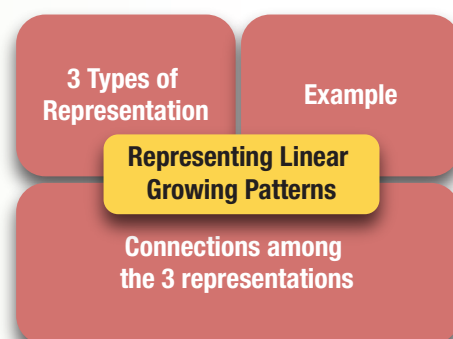
Ms. Willow wants her students to understand and use their strengths so she differentiates their learning based on this. She groups her students based on their area of confidence and has them join the centre whose task's initial representation is the one with which they are most confident. In addition, Ms. Willow pairs students, within the centres, based on their readiness and provides them with a task at an appropriate level of challenge. She addresses their readiness needs by providing parallel tasks – tasks of varying degrees of complexity, at each centre and by scaffolding her instruction as she works with various pairs of students in each of the centres. Ms. Willow has designed the centre activities to engage all of her students in several of the mathematical processes including representing, connecting, problem solving and communicating.

Parallel tasks, a tiering strategy, are designed with the same learning goals in mind, but are differentiated based on student readiness.

Ms. Willow observes the students as they work and poses questions to develop their algebraic thinking. While she has planned the tasks to appropriately challenge all students, she adapts them to respond in the moment to the learning needs of her students. For example, she notices that the one pair is having difficulty finding a pattern rule because their pictorial/concrete representation is not built in a way that the repeating unit in the growing part of the pattern can be recognized at each position. As a result, the students are unable to connect this to the position number. Ms. Willow does a think aloud using the 'Exploring Different Representations' digital tool (select 'Exploring Different Representations' at <http://oame.on.ca/mathies/learningTools.php>) with this pair of students. She asks the students to compare the pictorial representation for each position and discuss what stays the same and what changes. She then has the students rearrange their concrete model so that the repeating unit is recognized. Ms. Willow extends the task by having the students predict what the 100th position will look like and then write the pattern rule.

Two other pairs of students have quickly completed their tasks and are ready for an additional challenge, so Ms. Willow has the pairs exchange tasks to determine the pattern rule for the pictorial/concrete model of the other pair and then work together to pose and solve three extending questions (e.g., finding the term number for a given term, starting with a different initial representation) relating to the pattern. By adapting instruction as the needs arise, Ms. Willow is able to increase engagement and maximize the learning of all students.

To consolidate their understanding about the constant and changing parts of a linear relationship, students complete a graphic organizer in their centres and then discuss as a class – guided by Ms. Willow.



Leveraging Learning Preferences When Problem Solving (Mr. Young)

Mr. Young is helping his grade 9 Applied Mathematics class consolidate their learning related to problem solving and measurement for volumes of prisms, pyramids, cylinders, cones, and spheres. Earlier in the semester, to build their learning profiles, his students completed surveys to provide information about their learning preferences.

Brainstorming, as assessment for learning, provides opportunities for teachers to identify the mathematics readiness of their students.

Class learning profiles are useful tools to help teachers consider the characteristics of their students, including diverse learning preferences.

To begin the consolidation lesson, Mr. Young's students brainstorm to review the attributes of three dimensional (3D) figures and the relationships between them. Students select a preferred way to represent their thinking (e.g., poster, list, sticky notes) and form small groups to complete the brainstorming task. Mr. Young observes the students as they work, asking questions and gathering information about students' prior knowledge.

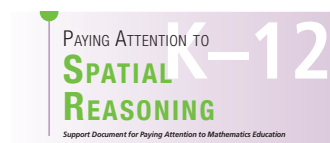
For today's class, Mr. Young prepares three learning centres (Data, People, and Things) to provide differentiated learning opportunities for his students related to solving volume problems for 3D figures. Students, having already completed a Data, People or Things preferences survey (www.gvltc.edu/data_people_or_things/), select their learning centre by deciding whether they prefer to work with people, interact with data, ideas and information or use 'things' such as materials and tools to solve measurement problems.

- At the Data centre, students create spreadsheets of the volume data using formulas and use trial and error to manipulate data to determine the dimensions of an example for each of the five 3D figures (prism, pyramid, cylinder, cone, and sphere) with a volume of 500 ml. The Data centre provides opportunities for students to work alone or with others.
- At the People centre, students work in pairs or triads to analyze and correct errors in solutions related to measuring the volume of each of the five 3D figures (prism, pyramid, cylinder, cone and sphere). Students use a digital tool (e.g., interactive white board or audio/video app) to record their thinking and work. The People centre encourages learners to work together and support each other's learning.
- At the Things centre, students pack 3D figures of one type (e.g., marbles) into rectangular prisms. They then use mathematics and calculations to justify the number they were able to pack. The Things centre provides students with a kinesthetic, hands-on approach.

While students engage at the centres, Mr. Young circulates, working with students to address gaps in prior knowledge as noted during their brainstorming and observing any misunderstandings the students may be having with the mathematics tasks. Mr. Young is pleased to see that most students are engaged in the learning, but he notices that Joey has moved from the People centre to the Data centre. Mr. Young reflects with Joey about his initial choice of learning centre and the reasons why he wishes to change to a different learning centre. He encourages Joey to continue his problem solving at the Data centre and arranges for Joey's partner to work with another pair in the People centre.

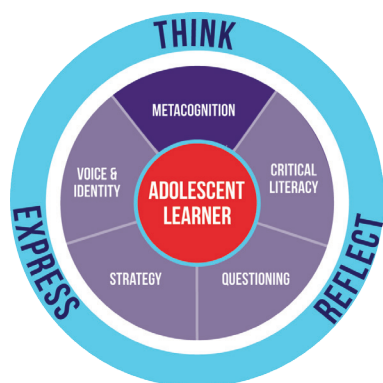
At the People centre, Mr. Young notices that a few students are having difficulty identifying errors in measurement. Mr. Young responds by working with this smaller group to review concepts using simpler problems. He demonstrates how to use manipulatives to make realistic estimates and check the reasonableness of the solutions. By adapting the instruction when needed, Mr. Young is able to respond effectively to learning preferences and mathematics readiness that surface during the lesson.

For another example of differentiation in Grade 9 Applied mathematics based on 'People, Data and Things' preferences, see Centennial SS video at: http://www.edugains.ca/newsite/di/dvd_secondary_success.html



<http://learnteachlead.ca/pdf/paying-attention-to-spatial-reasoning/>





Students demonstrate metacognitive thinking when they recognize how their attitudes, habits and dispositions influence the extent of their learning.

(Adolescent Literacy Guide, 2012)

The Specialist High Skills Major program incorporates differentiation by having students focus on their career interests. <http://www.edu.gov.on.ca/morestudentsuccess/program.html>

Pre-assessment (diagnostic) occurs before instruction begins so teachers can determine students' readiness to learn new knowledge and skills, as well as obtain information about their interests and learning preferences.

(Growing Success, p.31)



To consolidate learning, Mr. Young introduces new volume and measurement problems to the entire class. As students share possible solutions, he highlights different strategies (some developed at the learning centres) for solving each problem. Wanting his students to become more aware of themselves as learners and to be able to use metacognitive techniques to improve their learning, he asks his students to respond on a sticky note to the prompt: "How did your preferred way of working (with Data, People or Things) help you when solving the volume problem?"

Mr. Young is pleased with the way the centres motivated his students and helped them to consolidate their learning in math. He adds information about their 'People, Data, and Things' preferences to his class profile and encourages his students to note their preferences and what they have learned about themselves as math learners, in their Individual Pathways Plan (IPP).

The exit card is an effective strategy for self-assessment and will yield reflection and diagnostic information for future planning.

For more about Individual Pathways Plans, see *Creating Pathways to Success*:
<http://www.edu.gov.on.ca/eng/document/policy/cps/creatingpathwayssuccess.pdf>

"The experiences students have as they actively develop (their IPP) allow them to gather information about themselves and their opportunities; consider feedback from their teachers, parents and peers; make decisions and set goals and develop plans for achieving their goals."

(*Creating Pathways for Success*, p. 16)

Grade 11 University/College Mathematics

Engaging Students in Representing Functions (Ms. Bartraw)

Ms. Bartraw's Grade 11 University/College students have just finished graphing exponential functions to investigate how transformations of the graph affect the algebraic representation. The focus now is on modelling real world problems with appropriate types of functions – linear, quadratic or exponential.

On a pre-assessment that includes a task and a few 'interest' questions, Ms. Bartraw finds that the students vary in their interests as well as their readiness to model data with appropriate functions. Some of Ms. Bartraw's students are taking Specialist High Skills Major programs. Inspired by this, Ms. Bartraw decides to differentiate by having her students choose a mathematics problem based on an area of career interest. She sets up four stations, each with a problem based on a set of data, corresponding to one of the following areas: biology, economics, sports and agriculture.

The activity at each station begins with a set of discussion questions to help students share their knowledge and experience in the area of interest. As Ms. Bartraw observes students engaging in discussion, she notices two students not participating at the agriculture station. One, an English language learner, is unfamiliar with some of the vocabulary, such as acreage, and multiple meaning words such as sow, bluff and pen that are being used at the centre. Ms. Bartraw works briefly with the student using a tablet to provide visual support and an explanation of the vocabulary.

The other student at the agriculture station who is not participating is upset because his family farm has been shrinking in the last few years and as a result his family is struggling financially. Ms. Bartraw offers to help the student make connections between his situation and the math problem by considering it from various perspectives.

Differentiation on the fly can be as simple as changing a student's perspective or choice of task.

Group activity moves from the discussion to creating models for solving the problem. The students are to use their model to make predictions. Ms. Bartraw circulates and observes the students at the centres – particularly those who had not demonstrated readiness on the pre-assessment task. Some students are doing well and fully engaged in problem solving at the centre; a few, however,

from various centres, are struggling. Anticipating a need for review based on the pre-assessment, Ms. Bartraw has prepared some appropriate material to scaffold the learning for students so that their tasks have the same level of complexity as those of the other students. She has 6 students form their own group and uses a video (e.g., <http://m.youtube.com/watch?v=VS7R4cw6fa8>) and notes organizer to help them review concepts related to fitting exponential models to data. Students then work on an alternate task with the same learning goal as the centre tasks.

The group at the economics station stands out when a debate escalates. Half of this group is convinced that the data should be modelled with a linear function. The other half chooses an exponential model. Noticing that the linear model was developed using a small region of data, Ms. Bartraw listens to the discussion intently:

Jarod, arguing for a linear model: "Our linear model more accurately predicts the volume of the market at the three periods inside the data set and at this one period outside of the data set."

Nina, an exponential model advocate: "Your model might predict those points more accurately but if we extend it into the future it will absolutely be lower than the actual value."

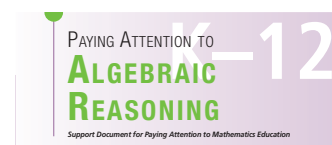
Ms. Bartraw has the economics group split and asks them to develop graphical and algebraic representations of their models and to re-examine how they fit – specifically considering the data on the far edges of the set. She also asks them to collaborate on mathematical arguments before presenting to the class. She recognizes sound reasoning in the students' justification for changing their models and feels it is important for students to evaluate their own models for errors.

In the end, when the groups present, Ms. Bartraw is pleased to see that the economics group has come to an agreement. Not only did they present their model and the method they used to arrive at it, they also highlighted the pitfall of using limited data to determine a model. This initiates discussion within the class on their data and each centre considers the implications of what an incorrect model may mean for each of the problems.

Scaffolding provides students with supports to build their knowledge and skills so that they are ready to learn or can tackle tasks at a particular level of complexity. As students develop in the particular area, supports are gradually removed.

Observation and frequent assessment should direct the actions at each step in the learning process.

STEP – Steps to English Language Proficiency is a framework for assessing and monitoring the language acquisition and literacy development of English language learners across The Ontario Curriculum. (http://www.edugains.ca/newsite/ell/step/english_second_language.html)



<http://learnteachlead.ca/projects/k-12-paying-attention-algebraic-reasoning/>

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A Complexity of Learning and Teaching Graphic

To address the diverse needs of all learners, educators consider varied components of instruction to refine their practice. Recent research (e.g., Leithwood, 2013; Hattie, 2012) investigating the learning and teaching process makes frequent reference to the importance of each of the components illustrated in the graphic below. Responsive, effective and coherent practice includes all of the following components:

REFLECTIVE STANCE that includes:

- Evidence-informed thinking about the** current state, the ideal state, how to bridge the gap, and how to gauge progress along the way
- Seeking feedback** on impact of teaching to inform next steps

LEARNING ENVIRONMENT that is culturally responsive and based on:

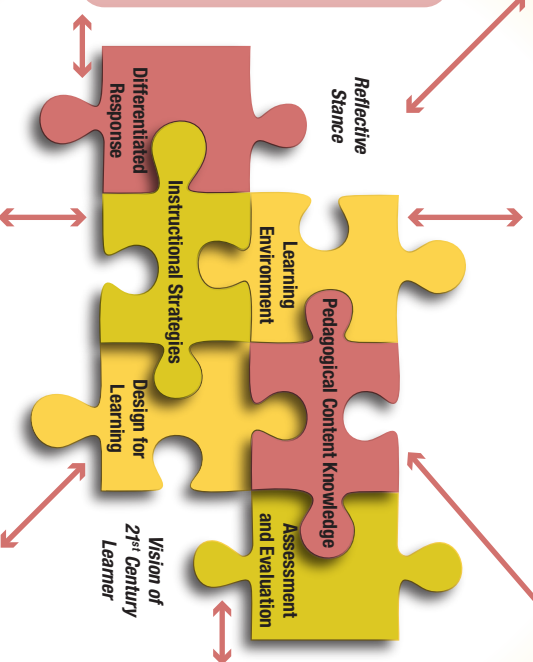
- A belief** that all students can learn
- An understanding of** 'how we learn'
- Our needs** as we learn: affirmation, relationships, challenge, contribution, power and autonomy, purpose, student voice

PEDAGOGICAL CONTENT KNOWLEDGE

Knowledge of the learner, curriculum and program within and across subject disciplines: content and pedagogy (e.g., organizing and connecting content, cognitive and metacognitive skills), and related learning skills and work habits

DIFFERENTIATED RESPONSE that addresses the needs of different learners:

- Know the Learner** – assess to gather information about student readiness, interests and/or learning preferences, and
- Respond** to the particular needs of different groups of learners by differentiating ways of learning and ways to demonstrate learning



ASSESSMENT to inform instruction and learning by identifying learning goals and success criteria, providing effective feedback as part of assessing for, as and of learning, and making thinking and learning visible

EVALUATION based on a variety of evidence (e.g., observations, conversations and products) documented over time and focusing on the achievement of overall expectations

INSTRUCTIONAL STRATEGIES that are engaging and strategic – based on where learning needs to go and targeted to the level of support required

Educators use a repertoire of instructional strategies that are responsive to student learning interests, preferences and assessed needs

Practice should be adaptive – selecting and fine-tuning strategies based on what works – for the student, the context and the intended area of learning

Educators ask themselves: Why this learning, with this student at this time, in this way? What is the impact?

DESIGN FOR LEARNING that is based on the concepts of universal design for learning and an understanding of how learning happens:

Focus and engage students by activating prior knowledge and experience, helping generate questions and discussing current understandings related to the learning goals

Engage in learning opportunities that are relevant, authentic and appropriate to needs, monitor progress, adapt or extend the learning with opportunities for shared, guided and independent practice

Consolidate, connect and reflect on the process as well as the new learning and understandings