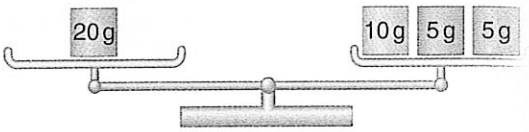
6.2 Using a Model to Solve Equations

Focus Solve equations by using a balance-scales model and verifying the solution.

Sometimes, systematic trial and inspection are not the best ways to solve an equation.

Balance scales can be used to *model* an equation. When pans are balanced, the mass in one pan is equal to the mass in the other pan.



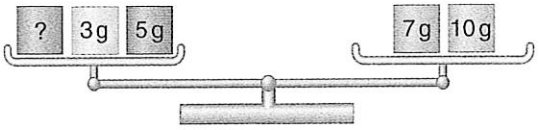
We can write an equation to describe the masses in grams.
$20 = 10 + 5 + 5$

Explore

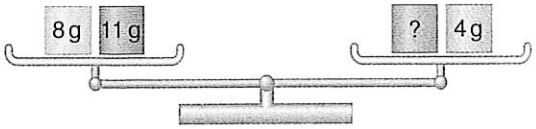
Use balance scales if they are available.
Otherwise, draw diagrams.

Here are some balance scales.
Some masses are known. Other masses are unknown.

Balance A



Balance B



➤ The pans are balanced.
For each balance scales:
- Write an equation to represent the masses.
- Find the value of the unknown mass.

➤ Make up your own balance-scales problem.
Make sure the pans are balanced and one mass is unknown.
Solve your problem.

Reflect & Share

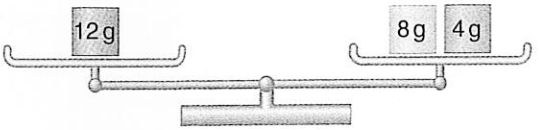
Trade problems with another pair of classmates.
Compare strategies for finding the value of the unknown mass.

Connect

We can use a balance-scales model to solve an equation.

When the two pans of scales are balanced,
we can adjust each pan of the scales in the same way,
and the two pans will still be balanced.

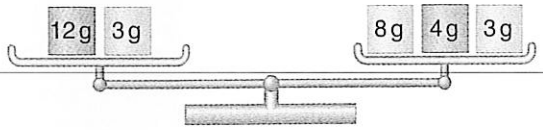
➤ Consider these balance scales:



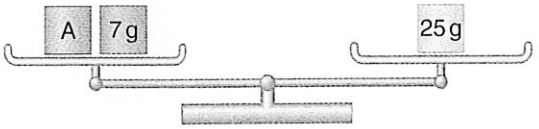
If we add 3 g to each pan,
the masses still balance.

$12\text{ g} + \mathbf{3\text{ g}} = 8\text{ g} + 4\text{ g} + \mathbf{3\text{ g}}$

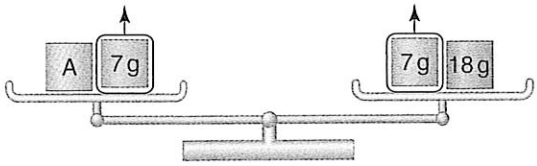
$15\text{ g} = 15\text{ g}$



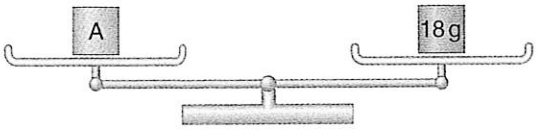
➤ Here is a balance-scales problem.
Mass A is an unknown mass.



If we remove 7 g from the left pan,
then Mass A is alone in that pan.
To keep the pans balanced,
we need to remove 7 g from the right pan too.
We replace 25 g with 7 g and 18 g;
then remove 7 g.



We are left with Mass A in the left pan
balancing 18 g in the right pan.
So, Mass A is 18 g.



We can *verify* the solution to this problem.
Replace Mass A with 18 g.
Then, in the left pan: $18\text{ g} + 7\text{ g} = 25\text{ g}$
And, in the right pan: 25 g
Since the masses are equal, the solution is correct.

To verify means to check the solution is correct.

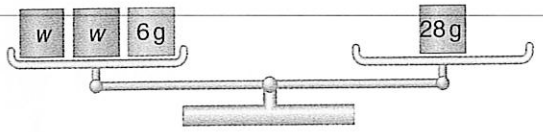
Example

A hockey team gets 2 points for a win,
1 point for a tie, and 0 points for a loss.
The Midland Tornadoes ended the season
with 28 points. They tied 6 games.
How many games did they win?
Write an equation you can use to solve the problem.
Use a model to solve the equation, then verify the solution.

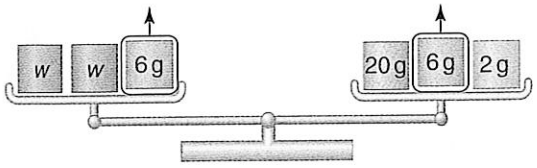
A Solution

Let w represent the number of games won by the Midland Tornadoes.
So, $2w$ represents the number of points earned from wins.
The team has 6 points from ties.
It has 28 points altogether.
So, the equation is: $2w + 6 = 28$

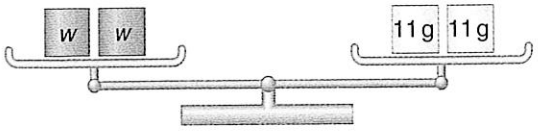
Use balance scales to model the equation.



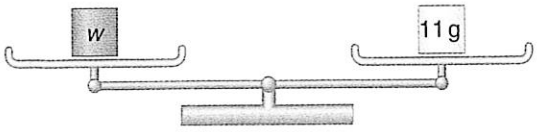
To get w on its own in one pan,
6 g has to be removed from the left pan.
So, select masses in the right pan
so that 6 g can be removed.
One way is to replace 28 g with 20 g + 6 g + 2 g.
Then, remove 6 g from each pan.



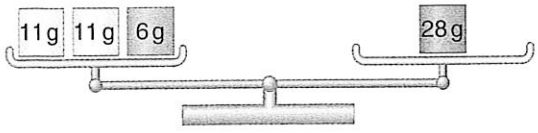
Two identical unknown masses
are left in the left pan.
20 g + 2 g = 22 g are left in the right pan.
Replace 22 g with two 11-g masses.



The two unknown masses balance
with two 11-g masses.
So, each unknown mass is 11 g.



The Midland Tornadoes won 11 games.
Verify the solution by replacing
each unknown mass with 11 g.
11 + 11 + 6 = 28, so the solution is correct.



The examples on pages 227 and 228 show these ways in which we preserve balance and equality:

- We can *add* the same mass to each side.
- We can *subtract* the same mass from each side.
- We can *divide* each side into the same number of equal groups.

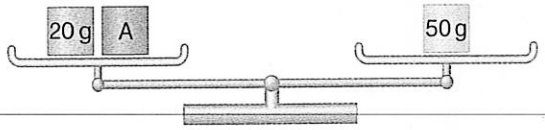
Later, we will show that:

- We can *multiply* each side by the same number by placing equal groups on each side that match the group already there.

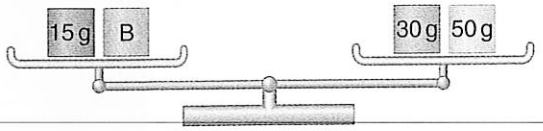
Practice

1. Find the value of the unknown mass on each balance scales. Sketch the steps you used.

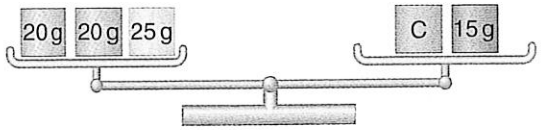
a)



b)



c)



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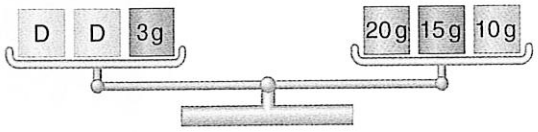


2. a) Sketch balance scales to represent each equation.

b) Solve each equation. Verify the solution.

i) $x + 12 = 19$ ii) $x + 5 = 19$

iii) $4y = 12$ iv) $3m = 21$

v) $3k + 7 = 31$ vi) $2p + 12 = 54$

3. a) Write an equation for each sentence.

b) Solve each equation. Verify the solution.

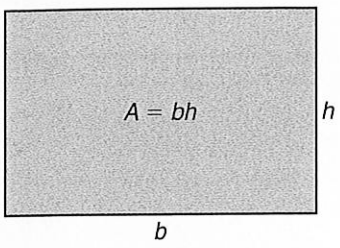
i) Five more than a number is 24.

ii) Eight more than a number is 32.

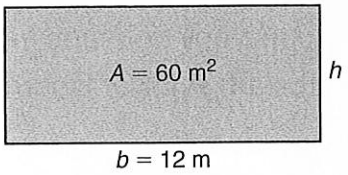
iii) Three times a number is 42.

iv) Five more than two times a number is 37.

4. The area of a rectangle is $A = bh$,
where b is the base of the rectangle and h is its height.
Use this formula for each rectangle below.
Substitute for A and b to get an equation.
Solve the equation for h to find the height.
Show the steps you used to get the answers.



a)



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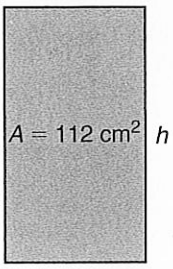

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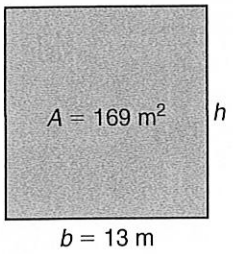

5. Assessment Focus Suppose the masses for balance scales are only available in multiples of 5 g.

a) Sketch balance scales to represent this equation:
$x + 35 = 60$

b) Solve the equation. Verify the solution.
Show your work.

6. Take It Further

a) Write a problem that can be solved using this equation: $x + 4 = 16$

b) How would your problem change if the equation were $x - 4 = 16$?

c) Solve the equations in parts a and b.
Show your steps.

7. Take It Further Write an equation you could use to solve this problem.
Replace the □ in the number 5□7 with a digit to make the number divisible by 9.

Reflect

Do you think you can always solve an equation using balance scales?
Justify your answer. Include an example.